

COMPARISON BETWEEN 2-D AND 3-D STIFFNESS MATRIX MODEL SIMULATION OF SASW INVERSION FOR PAVEMENT STRUCTURE

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ABSTRACT

The Spectral Analysis of Surface Wave (SASW) method is a non-destructive in situ seismic technique used to assess and evaluate the material stiffness (dynamic elastic modulus) and thickness of pavement layers at low strains. These values can be used analytically to calculate load capacities in order to predict the performance of pavement system. The SASW method is based on the dispersion phenomena of Rayleigh waves in layered media. In order to get the actual shear wave velocities, 2-D and 3-D models are used in the simulation of the inversion process for best fitting between theoretical and empirical dispersion curves. The objective of this study is to simulate and compare the 2-D and 3-D model of SASW analysis in the construction of the theoretical dispersion curve for pavement structure evaluation. The result showed that the dispersion curve from the 3-D model was similar with the dispersion curve of the actual pavement profile compared to the 2-D model. The wave velocity profiles also showed that the 3-D model used in the SASW analysis is able to detect all the distinct layers of flexible pavement units.

Keywords: 2-D and 3-D stiffness matrix model, SASW, pavement structure.

INTRODUCTION

The accurate information of the elastic modulus and thickness of the various pavement layers are needed for pavement management system. The Spectral Analysis of Surface Wave (SASW) is a non-destructive method that is based on the dispersion of Rayleigh waves that has been developed to determine the shear wave velocity corresponding to dynamic stiffness and thickness of each layer of the pavement profile. The SASW entirely operates from the surface of the pavement profile without needs of any drillings on the structure. The SASW method has been utilized in different application over the past decade from the advancement and improvement of the well-known steady-state technique which was initiated by Jones [1]. In the SASW analysis, the experimental dispersion curve of Rayleigh wave velocity versus wavelength is inverted to estimate the shear wave velocities and their variation with depth. Initially, the simple technique established by Heisey [2] was used to obtain the shear wave velocity profile in the SASW inversion. However, the technique is just sufficient for the profile with material stiffness increasing gradually with depth such as a normal soil profile.

For the pavement profile a more accurate approach, the stiffness matrix algorithm, is used to generate the theoretical dispersion curve for obtaining best fitting to the experimental dispersion curve. The purpose of this paper is to simulate and to compare the theoretical phase dispersion curve produced by the 2-D and 3-D stiffness matrix model which is employed in the SASW inversion process for the shear wave velocity assessment of an actual flexible pavement structures. Based on the shear wave velocities, the dynamic stiffness property (elastic modulus) of each layer in a pavement profile is also analyzed.

THEORY OF SPECTRAL ANALYSIS OF SURFACE WAVE

The SASW method is based on the particles motion of Rayleigh wave in heterogeneous media. The energy of Rayleigh waves from the source propagate mechanically along the surface of media and their amplitude decrease rapidly with depth. Particle motions associated with Rayleigh wave are composed of both vertical and horizontal components, which when combined, formed a retrogressive ellipse close to the surface. In homogenous, isotropic, elastic half-space, Rayleigh wave velocity does not vary with frequency. However, Rayleigh wave velocity varies with frequency in layered medium where there is a

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variation of stiffness with depth. This phenomenon is termed dispersion which the frequency is dependent on Rayleigh wave velocity. The ability of detecting and evaluating the depth of the medium is influenced by the wavelength and the frequency generated. The shorter wavelength of high frequency penetrates the shallower zone of the near surface and the longer wavelength of lower frequency penetrates deeper into the medium.

The range of wavelength to be used as a guide for the receiver spacing can be estimated from the shear wave velocities of the material anticipated at the site from:

$$\lambda = \frac{V_s}{f} \quad (1)$$

where f is the frequency and V_s is shear wave velocity predicted from soil classified at the site or the initial information from other site investigation works. The high and low frequency waves groups needed can be generated by various transient sources with different weights and shapes. Rosyidi [3] and Rosyidi et al. [4] studied and reported the effect of various weights and shapes of sources to generate the appropriate wave frequencies.

The experimental dispersion curve of phase velocity and wavelength may be developed from phase information of the transfer function at frequency ranges satisfying the coherence criterion. In addition, most of the researchers apply the Heisey's filtering criteria [2] with a wavelength greater than $\frac{1}{2}$ and less than 3 of the receiver spacing which is able to capture an effective part of the individual dispersion. A travel time between the receivers for each frequency can be calculated by:

$$t(f) = \frac{\phi(f)}{360f} \quad (2)$$

where f is the frequency, $t(f)$ is the travel time and $\phi(f)$ is the phase difference in degrees at a given frequency. The distance of the receiver (d) is a known parameter. Therefore, Rayleigh wave velocity (phase velocity) at a given frequency (V_R) is simply obtained by:

$$V_R = \frac{d}{t(f)} \quad (3)$$

and the corresponding wavelength of the Rayleigh wave, L_R may be written as:

$$L_R(f) = \frac{V_R(f)}{f} \quad (4)$$

ANALYTICAL MODELING

The actual shear wave velocity of the pavement profile is produced from the inversion of the composite experimental dispersion curve. In the inversion process, an automated modeling analysis is used where a set of homogeneous layer profile extending to infinity in the horizontal direction is assumed. The last layer is usually taken as a homogeneous half-space. Based on the initial profile, a theoretical dispersion curve is modeled using the stress wave propagation theory and employing the dynamic stiffness matrix method [5]. Modeling of the SASW analysis can be accomplished with two approaches: considering only plane generalized Rayleigh waves that is called the two dimensional model, or calculating for three directions of wave propagation effect, called the three dimensional model. In the 2-D approach, the Rayleigh wave is assumed as simple plane wave which horizontally propagates along the surface of a multi-layered medium in x direction and vertically penetrates into the medium (Figure 1). The plane in the x - y axis is the boundary of the elastic half space and z is positive downward. The Rayleigh wave displacement of the 2-D model is calculated in the direction of x and z while it is independent in y direction. Whereas, the 3-D model simulates the Rayleigh wave traveling along the surface in radian function which is expressed in cylindrical coordinates and decaying with the depth as a function of z direction. The dynamic response in the pavement profile is measured on the surface at distance r from an impact source which is a vertical circular disk loaded at the center of the system (Figure 1). The displacement of Rayleigh wave propagation is described in the spatial domain and energy transmission in the radial direction due to vertical oscillation of a circular disk on the surface of medium. Using *Fourier* transform and phase different method (Equation 2 to 4), the displacement of Rayleigh wave in spatial domain from the 2-D and 3-D model is then simulated to the phase velocity in frequency domain which is usually called as a theoretical dispersion curve. In the SASW analysis, a theoretical dispersion curve is ultimately matched with the experimental dispersion curve in the lower Root Mean Square (RMS) error of an optimization technique [6].

Two-Dimensional Model

The aim of this model is to obtain the theoretical dispersion curves for plane Rayleigh waves propagating in a horizontally layered pavement profile with known properties such as mass density (ρ), Poisson ratio (μ), elastic modulus (E), shear modulus (G) and damping ratio (D) which is shown in Figure 1. The differential equations of the stress wave

propagation allows the stresses and displacements at the top of the layer of the pavement profile to be related to those at the bottom as a function of vibration and the wave number (or wavelength) through a dynamic stiffness matrix [5]. The stiffness matrices of the individual layers (as well as that of a half space) can be assembled following the normal rules of matrix structural analysis leading to a system equations of the form:

$$P = K U \tag{5}$$

where K is the dynamic stiffness matrix of the complete pavement materials, U are the amplitudes of the displacements at the various layer interfaces (starting at the free surface) and P is the vector of applied loads at these interfaces. The terms of K are function of the frequency, ω , and the wave number, k. In order to have a non trivial displacement without any external loads the determinant of K must be equal to 0. The terms of the matrix K is a tridiagonal matrix of 2×2 submatrices and the determinant can be easy evaluated for any value of k and a fixed ω .

The solution technique for arbitrary loading requires the time history of the specified load forces to be decomposed into its different frequency using a Fourier transform. For each frequency, any given load vector is expanded in a double integral Fourier transform in the two horizontal directions for Cartesian coordinates as follows:

$$\bar{P}(k, z, \omega) = \iint_{-\infty}^{+\infty} P(x, z, t) e^{-i\omega x} dt dx \tag{6}$$

where \bar{P} is given load vector in the wave number domain. The displacement in the wave number domain is then solved by:

$$\bar{U}(k, z, \omega) = K^{-1} \bar{P}(k, z, \omega) \tag{7}$$

The actual displacements are computed as:

$$U(x, z, t) = \frac{1}{2\pi} \int \int_{-\infty}^{+\infty} \bar{U}(k, z, \omega) e^{ikt} dk d\omega \tag{8}$$

where U is the actual displacement in time-space domain.

Three-Dimensional Model

In the 3-D model, displacements and stresses (or traction) on a horizontal surface is expanded in Fourier series in the circumferential direction and in terms of cylindrical function (Bessel, Neuman or Hankel functions) in the radial direction as shown by Kausel and Rösset [5]. For application to cases with as axisymmetric loading only one term of Fourier series is needed (the 0 term), and the radial and vertical displacements (U and W) can be expressed by:

$$U(r) = qR \int_{k=0}^{\infty} \bar{u} J_1(kR) J_1(kr) dk ; \text{ and } W(z) = qR \int_{k=0}^{\infty} \bar{w} J_1(kR_0) J_0(kr) dk \tag{9}$$

where J_0 and J_1 are the zero and the first order Bessel function, k is the wave number, r is the radial distance from the source, R is the radius of the disk and q is the magnitude of the uniformly distributed load. \bar{u} , and \bar{w} are functions of k and can be obtained finding the solution to Equation 5 for a harmonic load at the surface with wave-length $2\pi/k$.

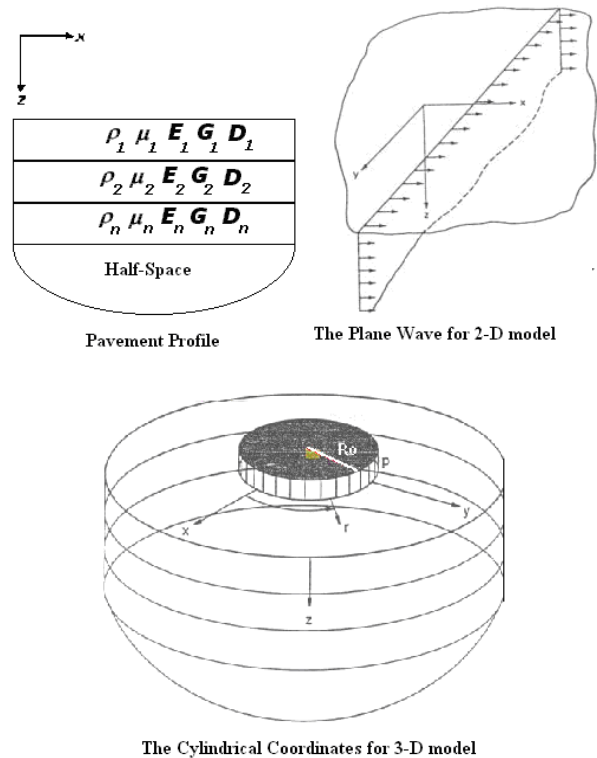


Figure 1. The Plane Wave Propagation for 2-D Model and the Cylindrical Coordinate for 3-D Model Concept

The solution of the problem requires assembling the dynamic stiffness matrix of the pavement profile, solving it for various values of k (wave number) and evaluating the integrals of Equation 10. The computation is relatively time consuming where there is a large number of layers and the integration requires special precautions if there is no damping in the pavement materials. An alternative solution can be obtained by expanding the terms of the dynamic stiffness matrix of a layer in terms of k and keeping only up to second degree items. The stiffness matrices of each layer and of the total profile can be expressed in the form:

$$K = A k^2 + B k + C - \Omega^2 M \tag{10}$$

where the solution for the matrices A, B, C and M can be founded in [5]. The numbers and in-plane

modes of propagation are now the solution of a quadratic eigenvalue problem. Kausel and Rössset [5] had shown that the displacements \bar{u} , and \bar{w} , in Equation 9, can be written as:

$$\begin{aligned} \bar{u} &= \sum_{i=1}^{2n+2} u_{il} w_{il} \frac{k}{k_i(k^2 - k_i^2)}; \text{ and} \\ \bar{w} &= \sum_{i=1}^{2n+2} w_{il}^2 \frac{k}{k_i(k^2 - k_i^2)} \end{aligned} \quad (11)$$

For a system of n layers over a half-space, where u_{i1} and w_{i1} denote the horizontal and vertical displacement at the surface in the i^{th} mode, can be found from the corresponding mode shape. By substituting Equation 11 to 9, the integral can be evaluated analytically in closed form [5]. This solution is particularly convenient when dealing with a large number of layers as in the case when it is desired to obtain a detailed variation of pavement properties.

PAVEMENT MATERIAL STIFFNESS

The dynamic elastic modulus of the pavement material can be determined using the shear wave velocities from the following equations [7]:

$$E = 2 \frac{\gamma \cdot V_s^2}{g} (1 + \nu) \quad (12)$$

where E is the dynamic elastic modulus, ν , the Poisson ratio, V_s , shear wave velocities, g, gravitational acceleration and γ , total unit weight of the material. Rosyidi [3] has explained that the modulus parameter of material is constant below a strain of about 0.001% and is equal to the maximum level. In this strain range, the material is typically independent of strain amplitude [8].

EXPERIMENTAL WORK OF SASW AND DCP TEST

An impact source on a pavement surface is used to generate Rayleigh waves at a point. These waves are detected with two acceleration receiver locations and the signals are recorded using a spectrum analyzer for post processing (Figure 2). Several configurations of the receiver and the source spacing test are required in order to sample different depths. Short receiver spacing of high frequency (short wavelength) are used to sample shallow layers while long receiver spacing of low frequency (long wavelengths) is used in sampling deeper layers. For detecting the pavement profile, the receiver spacings of 5, 10, 20, 40, 80 and 160 cm were implemented. The high frequencies of the sources were generated from the ball bearing weight of 5 to 20 g while the lower

frequencies were produced from hammer weight of 3 to 5 kg. All the data collected are transformed using fast Fourier transform (FFT) to frequency domain by a spectrum analyzer. Two functions in frequency domain between the two receivers are of great importance: (1) the coherence function, and (2) the phase information of the transfer function spectrum. The coherence function is used to visually inspect the quality of signals being recorded in the field. The transfer function spectrum is used to obtain the relative phase shift between the two signals in the range of the frequencies being generated as shown in Figure 3. By repeating the procedure outlined by Equation (2) through (4) for each frequency value, the Rayleigh wave velocity corresponding to each wavelength is evaluated and the experimental dispersion curve is generated.

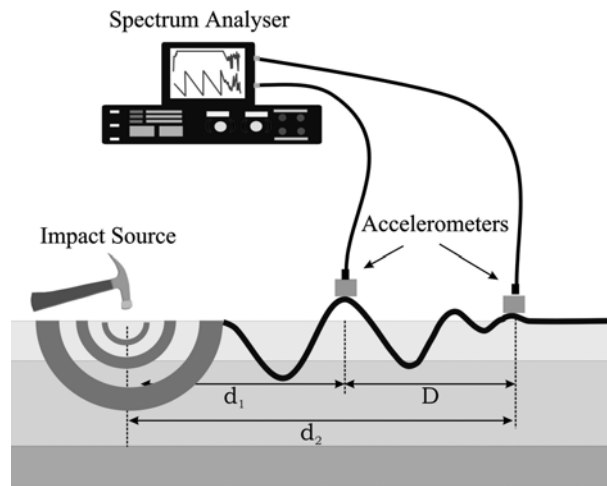


Figure 2. Field Measurement Configuration

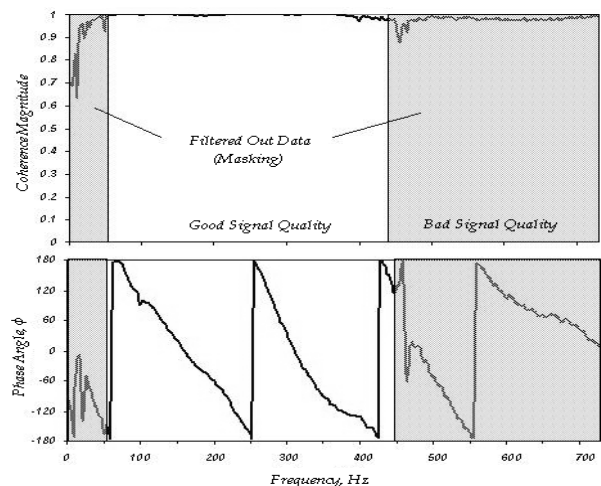


Figure 3. The coherence and the Transfer Function Spectrum

For validating the result in this study, the Dynamic Cone Penetrometer (DCP) test was also conducted in the same location of SASW test in order to obtain the analytic elastic modulus of the pavement materials using Shell's formulation. The dynamic cone penetro-

meter (DCP) is also employed as a rapid means of assessing the sequence, thickness and the in-situ bearing capacity of the unbound layers and the underlying subgrade of the pavement structure. The DCP uses a 8 kg steel mass falling 20 inches (50.8 cm) striking an anvil causing a penetration of 1.5 inches (3.8 cm) from a cone with a 60° vertex angle seated in the bottom of a hand augered hole. The blows required to drive the embedded cone a depth of 1-3/4 inches have been correlated to N values of the Standard Penetration Test (SPT). The depth of cone penetration is measured at selected penetration or hammer-drop intervals, and the soil shear strength is reported in terms of DCP index. The relationship between the DCP and the field California Bearing Ratio (field-CBR) can be determined using the model derived by Klyen and van Heerden [9]. The result obtained in their study can be written as follows:

$$\log \text{CBR} = 2.628 - 1.273 \log (\text{DCP}) \quad (13)$$

where DCP is the penetration mm/blows. The elastic modulus of pavement material can be calculated using Shell' formulation [10] as follows:

$$E_{\text{dynamic}} = 1,500 \text{ CBR (in Psi)} \quad (14)$$

RESULTS AND DISCUSSIONS

In order to show the dispersion result obtained from the 2-D and 3-D analysis, the actual pavement profile was studied. The road profile consists of an asphalt layer (70 mm thick), and a base of crushed aggregate (53 mm thick) over a subgrade. Thicknesses of the pavement layers are shown in Figure 5. The experimental dispersion curve of actual pavement profile was constructed using the experimental data from SASW test which is presented in Figure 4. In order to simulate the 2-D and 3-D model of dispersion curve, the starting model parameters (SMP) were then generated. The starting model parameters are employed to facilitate the inversion process and to avoid the convergence to a local minimum in the inversion problems ([3] and [6]). The starting model parameters are constructed using the required information which include Poisson's ratio, mass density, the shear wave velocities, material damping ratio and layer thicknesses. If the above information from the previous testing in the site are not available, they can be obtained with reasonable assumption. The starting model parameters of a pavement profile was divided into three layers of pavement structure, each layer was still separated into smaller layers for a detailed model of a pavement profile. The pavement layer properties of the starting model parameters are presented in Table 1.

Table 1. The Starting Model Parameters for Generating 2-D and 3-D Model

Layer Properties for	No. Layer	Thickness [m]	Shear Wave Velocity [m/s]	Mass Density [kg/m ³]	Poisson	Damping Ratio
Surface Course	1	0.055	871.17	2320	0.33	0.02
	2	0.040	947.36	2320	0.33	0.02
Base Course	3	0.040	453.47	2320	0.33	0.02
	4	0.050	151.84	2320	0.33	0.02
	5	0.050	151.84	2000	0.33	0.02
	6	0.050	151.84	2000	0.33	0.02
	7	0.050	151.84	2000	0.33	0.02
	8	0.100	151.84	1900	0.33	0.02
	9	0.100	151.84	1900	0.33	0.02
	10	0.100	257.42	1900	0.33	0.02
Subgrade	11	0.200	346.24	1900	0.33	0.02
	12	0.200	269.50	1900	0.33	0.02
	13	0.500	188.19	1900	0.33	0.02
	14	0.500	165.21	1900	0.33	0.02
	15	1.000	86.08	1900	0.33	0.02
	16	1.000	82.60	1900	0.33	0.02
Half-Space	17	403.500	82.60	1900	0.33	0.02

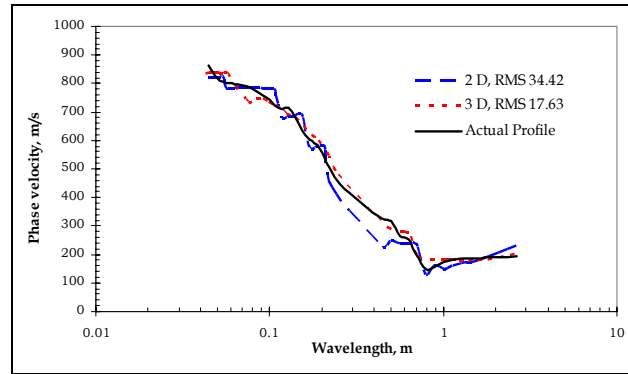


Figure 4. Comparison Between the 2-D, 3-D and the Experimental Dispersion Curve

Based on the starting model parameters, the theoretical dispersion curve was simulated using 2-D and 3-D stiffness matrix algorithm. Both theoretical curves were then compared to the experimental data and modifying the assumed properties to improve the fit. Figure 4 shows the theoretical dispersion curve of the 2-D and 3-D model compared to the experimental dispersion curve of the actual pavement profile. The result of 2-D model with the first mode appears to be in good agreement with the experimental data for wavelength up to 0.2 m. The phase velocities obtained with 3-D model are in better agreement with the experimental data. The phase velocity from 3-D model almost agrees well to all of wavelengths. It can be validated with the error measurement of root-mean-square (RMS) which is 34.42 and 17.63 m/s for 2-D and 3-D model respectively. It shows that the 3-D model of the dispersion curve is more fitted to actual profile than the 2-D model.

The result of inversion process illustrates that the shear wave velocities versus depth profile of the 3-D model shows clearly the difference of the interface between each layer of pavement profile as shown in Figure 5. The figure also shows that the 2-D shear

wave velocities profile is extremely different with the 3-D profile. The 2-D model can not successfully detect the interface between the asphaltic and road base layer. It can be explained that when the modulus of underlying layer is smaller than upper layer (pavement profile), the propagation of surface waves are recorded at maximum frequency (higher mode) which there are no real wave numbers occurred. It is caused by interferences of the refraction and refraction of the body wave in the interface. Consequently, it needs the determining the complex eigenvalues leading also to complex phase velocity of surface waves. The 2-D model does not provide algorithm for above case.

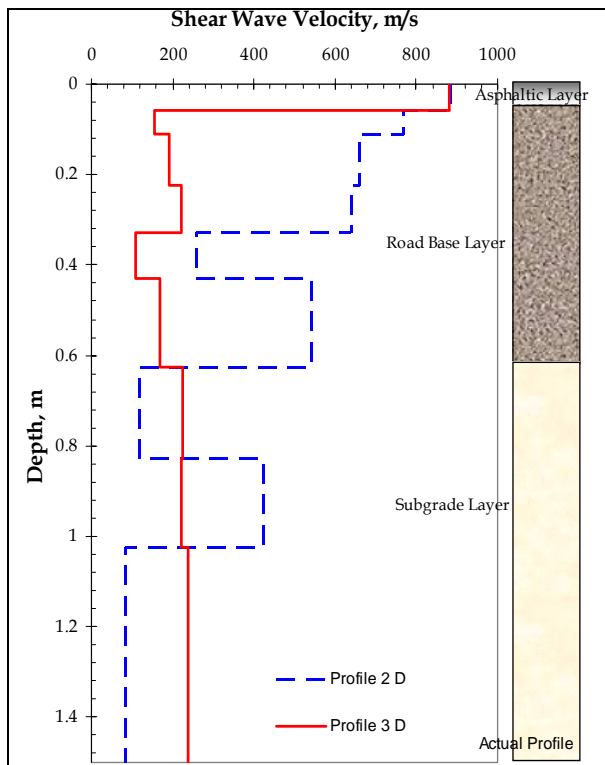


Figure 5. Comparison the Shear Wave Velocities Between the 2-D and 3-D Model

Table 2 shows the average dynamic elastic modulus that is calculated from the shear waves profile of the 2-D and 3-D using Equation 12 compared to the elastic modulus of the actual pavement profile from the DCP test. The difference of the elastic modulus between the 3-D model with the analytical result for road base is 6.45 % and for subgrade layer is 1.45% which are better than the 2-D model result.

Table 2. Comparison the Elastic Modulus Between the 2-D and 3-D Model of SASW Measurement and the DCP test-Shell's Analytical Formulation

Pavement Layer	Depth m	E _{SASW} (MPa)		E _{analytic} (MPa)	Difference (%)	
		2-D	3-D		2-D	3-D
Asphalt	0 - 0.07	4787.126	4808.038	---	---	---
Road Base	0.07 - 0.6	1504.570	343.377	365.521	75.71%	6.45%
Subgrade	> 0.6	122.471	137.588	135.588	10.71%	1.45%

CONCLUSIONS

The theoretical dispersion curve of the 2-D and 3-D dynamic stiffness matrix model was simulated and compared in the SASW inversion process for pavement structures. The 3-D model produces a better theoretical dispersion curve compared to the 2-D model. Compared to the actual pavement profile the 3-D model gives an RMS value of 17.34 m/s. The use of the 3-D model can produce the true wave propagation phenomena involved in the SASW test for irregular stiffness profile with sharp changes in the properties of the layers such as most pavement structure. Good agreement is also obtained in the measurement of dynamic elastic modulus using the 3-D model SASW inversion as compared to the actual profile from the DCP test. Ultimately, this study shows that the capability of the 3-D dynamic stiffness model to be used in the inversion process for characterizing the stiffness of the pavement layers in terms of shear wave velocity for pavement design and evaluation.

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