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Spline Nonparametric Regression Analysis of Stress-Strain Curve of Confined Concrete

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Abstract: Due to enormous uncertainties in confinement models associated with the maximum compressive strength and ductility of concrete confined by rectilinear ties, the implementation of spline nonparametric regression analysis is proposed herein as an alternative approach. The statistical evaluation is carried out based on 128 large-scale column specimens of either normal- or high-strength concrete tested under uniaxial compression. The main advantage of this kind of analysis is that it can be applied when the trend of relation between predictor and response variables are not obvious. The error in the analysis can, therefore, be minimized so that it does not depend on the assumption of a particular shape of the curve. This provides higher flexibility in the application. The results of the statistical analysis indicates that the stress-strain curves of confined concrete obtained from the spline nonparametric regression analysis proves to be in good agreement with the experimental curves available in literatures.

Keywords: confinement model, ductility, spline nonparametric regression analysis, stress-strain curves.

Introduction

The regression analysis has been playing an important role in the theory of approximation and statistics for many years. It is used to evaluate the effects of the independent to the dependent variables by observing the trend of relation between the two types of variables. The evaluation can be carried out using two approaches, i.e. the parametric approach, which is frequently used in practice, and the nonparametric approach.

The use of piecewise polynomial has been increasingly popular due to its flexible nature. It is effective in handling a function or a set of data comprising the local natures [1,2]. One of the essential piecewise polynomial is a spline polynomial. The application of the spline nonparametric regression analysis has been widely found in many fields, such as in economics [3] and medicine [4]. However, the implementation of this type of analysis in civil engineering, particularly in reinforced concrete area, is still seldom and not popular.

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The spline polynomial has many beneficial statistical properties for use in the regression analysis [1,5]. It is a piecewise polynomial which has segmental properties. The segmental properties of the spline polynomial give higher flexibility in the application compared to the ordinary polynomial. This allows the spline polynomial effectively adapting itself with the local characteristics of a function or a set of data.

To date, the regression analysis, which is frequently used by the researchers to obtain the stress-strain curve of confined concrete, is the parametric regression analysis, mostly linear or quadratic. Since it is very simple, the resulting stress-strain curves of confined concrete still have appreciable discrepancies with the experimental curves, particularly in term of ductility along the descending branch. This causes very significant uncertainties in the confinement model of concrete under uniaxial compression.

Problems of confined concrete have long been recognized and investigated both experimentally and analytically in the past [6]. Several confinement models have been developed to predict the stress-strain curve of normal- as well as high-strength concrete. All proposed models are based on previous experimental work where several parameters have been derived from the parametric statistical processing of the results. However, the evaluation of these models against other models was based on limited number of specimens and model parameters. Furthermore, the confinement models for predicting the stress-strain curve of confined concrete proposed by various researchers are applicable only for normal- or high-strength concrete. Hence, it is deemed necessary to propose an alternative analytical model which can be used for predicting the actual stress-strain curve of confined concrete of either normal- or high-strength concrete.

The present investigation focuses only those models developed by the statistical analysis for both normal- and high-strength concrete square columns confined by rectilinear normal and high yield steel ties. All the well-documented analytical models proposed so far are applied to predict the results of experimental tests on large-scale specimens carried out by various researchers [6,7,8]. A database was compiled, including a total of 128 square column specimens from tests involving monotonic axial compression loading. The concrete strength considered ranges from about 25 to 125 MPa, while the yield strength of lateral steel (f_{yh}) ranges from about 250 to 1390 MPa. The volumetric ratio of transverse reinforcement (ρ_s) calculated with respect to the centerline of the perimeter tie ranges from 0.8 and 5 percent. The test data were evaluated by using the spline nonparametric regression analysis. The proposed statistical model can be applied for both confined normal- and high-strength concrete, as well as for either normal or high yield steel ties.

Nonparametric Regression

The parametric regression analysis uses a particular shape of regression curve. If there is no information on the shape of regression curve $f(t_i)$, the use of the nonparametric regression approach is recommended [2,9]. The approach does not depend on a particular shape of regression curve. It provides higher flexibility compared to the parametric regression approach.

The regression function $f(t_i)$ of a nonparametric regression approach is assumed to be smooth as contained within the Sobolev's space $W_2^m[0,1]$ [2,9]. The nonparametric regression model is formulated as follows:

$$y_i = f(t_i) + \varepsilon_i;\ i = 1, 2, \ldots, n;\ 0 \le t_i \le 1 \tag{1}$$

where i is the number of the samples; n is the sample number; y_i is the response variable; ε_i is the random error; and t_i is the predictor variable.

The spline approach has a functional basis. The functional basis normally used is the truncated spline and B-spline [10]. In this paper, the truncated spline approach is selected for the nonparametric regression analysis to evaluate the confinement parameters.

Spline Polynomial

The use of spline polynomial in the regression analysis was first introduced by Whittaker (1923) [1,5], whereas the use of spline in the optimization problems was first developed by Reinch (1967) [1,5]. The truncated spline nonparametric regression analysis is one of the alternatives of nonparametric regression analyses.

The nonparametric regression model can be written in a general form given by Eq. (1). If the truncated spline polynomial is implemented in a nonparametric regression analysis, the regression function f (t) can be rewritten as follows:

$$f(t) = \sum_{i=0}^{k-1} \alpha_i t^i + \sum_{j=1}^{n} \beta_j (t - u_j)_+^{k-1} \tag{2}$$

where α_i and β_j are the real constants; $i = 0, 1, \ldots, k - 1$ and $j = 1, 2, \ldots, n$; and

$(t-u_j)_+^{k-1} = (t-u_j)^{k-1}$, for $t \ge u_j$

$(t-u_j)_+^{k-1} = 0$, for $t < u_j$

in which u_j is the knotty points; $j = 1, 2, ..., n$; and the value of $k - 1$ indicates the degree of spline polynomial. The spline polynomial above has the following natures:

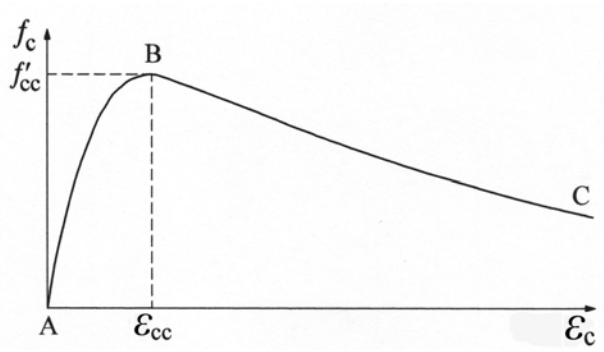
a) f is a $(k-1)$-degree piecewise polynomial within the interval of $[u_j, u_{j+1}]$;
b) f has a $(k-2)$-degree continuous derivative;
c) $f^{(k-1)}$ is a piecewise function with knotty points $u_1, u_2, \ldots, u_n$.

The values of $k = 2$, 3, and 4 define the spline polynomials as linear, quadratic, and cubic spline polynomials, respectively.

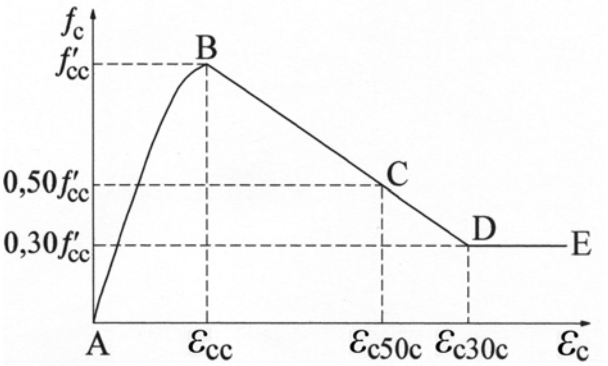
Overview of Confinement Models

The analytical models proposed by the following researchers are studied: Fafitis and Shah (FS) [6]; Kappos and Konstantinidis (KK) [7]; and Faimun, Aji, Tavio, and Suprobo (FATS) [8]. The stress-strain curves of the reviewed analytical models are illustrated in Fig. 1, while the constitutive equations of the models are listed in Table 1.

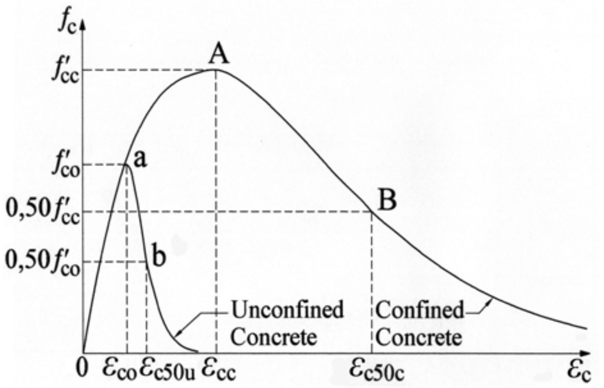
All models are using a parabolic form for the ascending branch. All those models have been proven to give reasonably good predictions on the ascending branch, particularly in term of the increase in concrete strength. The descending branch of analytical models, however, showed an uncertainty in predicting the actual ductility of confined concrete [6,7,8].



(a) Fafitis and Shah (FS) model



(b) Kappos and Konstantinidis (KK) model



(c) Faimun, Aji, Tavio, and Suprobo (FATS) model

Fig. 1. Analytical stress-strain curves for confined concrete

Proposed Empirical Model for Confined Concrete

The ascending branch of the curve is using an equation originally proposed by Popovics [13], by replacing f'_c for plain concrete with f'_{cc} for confined concrete. This equation provides an ascending branch, which is in close agreement with the test curves. The statistical evaluation summarized in the subsequent discussion shows that in overall the descending branch proposed originally by Cusson and Paultre [12] gives close agreement for all the key parameters. The equations by Faimun et al. [8], as given in Table 1, were then used as the basis for developing a modified model better fitting the empirical stress-strain curves of confined normal- as well as high-strength concrete. Since the modulus of elasticity given by Faimun et al. [8] is only valid for high-strength concrete, the more general modulus of elasticity of concrete (E_c), which is applicable for either normal- or high-strength concrete, is adopted in the proposed model. The equation was adopted earlier by Kappos and Konstantinidis [7] from Ref. [14], and given as follows:

$$E_c = 22{,}000\left(\frac{f'_c}{10}\right)^{0.3} \text{ (MPa)} \quad (3)$$

The equation that relates between the strength gain of confined concrete $f'_{cc} - f'_{co}$ and the effective capacity of transverse reinforcement $K_e\rho_s f_{yh}$ is obtained by the spline nonparametric regression analysis using the experimental data, given as follows:

$$\hat{y}_1 = 2.52 + 2.63x_1 - 0.06x_1^2 + 0.05\,(x_1 - 13.5)_+^2 \quad (4)$$

in which, $(x_1 - 13.5)_+^2 = (x_1 - 13.5)^2$, if $x_1 \geq 13.5$

$(x_1 - 13.5)_+^2 = 0$, if $x_1 < 13.5$

where $\hat{y}_1 = f'_{cc} - f'_{co}$, and $x_1 = K_e\rho_s f_{yh}$. f'_c is the compressive strength of standard cylinder, and K_e is the modified Sheikh and Uzumeri [15] factor for calculating the effectiveness of confinement given by the following formula:

$$K_e = \left(1 - \frac{\sum w_i^2}{6b_c d_c}\right)\left(1 - \frac{s}{2b_c}\right)\left(1 - \frac{s}{2d_c}\right) \quad (5)$$

Description on each parameter used in Eq. (5) is shown in Fig. 2.

A spline nonparametric regression analysis was performed using the experimental results to express the relationship between the normalized strain gain at peak stress $(\varepsilon_{cc}/\varepsilon_{co}) - 1$ and the effective capacity of transverse reinforcement $K_e\omega_w$. The best fit parabolic curve passing through the data points is given by Eq. (6).

Table 1. Summary of models and constitutive equations for stress and strain of rectilinearly confined concrete

Model	Equations	Comments
Fafitis and Shah [6]	$f_c = f'_{cc}\left[1-\left(1-\frac{\varepsilon}{\varepsilon_{cc}}\right)^A\right]$ $\quad f'_{cc} = \lambda_2\left[f'_c + \left(1.15+\frac{21.02}{f'_c}\right)f_{lat}\right]$ $f_c = f'_{cc}\exp\left[-K(\varepsilon-\varepsilon_{cc})^{1.15}\right]$ $\varepsilon_{cc} = 1.027\times10^{-7} f'_c + 0.0296\frac{f_{lat}}{f'_c} + 0.00195$ $K = 0.17 f'_c \exp\left(\frac{-0.01 f_{lat}}{\lambda_1}\right)$ $\quad f_{lat} = \frac{A_{sh} f_{yh}}{s d_e}$ $\quad A = E_c\frac{\varepsilon_{cc}}{f'_{cc}}$ $\lambda_1 = 1+25\frac{f_{lat}}{f'_c}\left[1-\exp(-0.0223 f'_c)^9\right]$ $\quad k = 24.65 f'_c e^{\left(-1.45\frac{f_{lat}}{\lambda_1}\right)}$ $\lambda_2 = 1+15\left(\frac{f_{lat}}{f'_c}\right)^3$ $\quad \varepsilon_{c50c} = \varepsilon_{cc} + \left(\frac{-\ln 0.5}{k}\right)^{\frac{1}{1.15}}$	f'_{cc}, ε_{cc} derived from statistical analysis of experimental data on 76 × 152 mm cylinders with 26 ≤ f'_c ≤ 66 MPa
Kappos and Konstantinidis [7]	$f_c = \frac{f'_{cc}\frac{\varepsilon}{\varepsilon_{cc}}\frac{E_c}{E_c-E_{ci}}}{\frac{E_c}{E_c-E_{ci}}-1+\left(\frac{\varepsilon}{\varepsilon_{cc}}\right)^{\frac{E_c}{E_c-E_{ci}}}}$ $\quad f'_{cc} = f'_{co} + 10.3(k_e\rho_s f_{yh})^{0.4}$ $f_c = f'_{cc}\left[1-0.5\frac{\varepsilon-\varepsilon_{cc}}{\varepsilon_{c50c}-\varepsilon_{cc}}\right] \ge 0.3 f'_{cc}$ $\quad \varepsilon_{co} = \frac{0.7(f'_c)^{0.31}}{1000}$ $E_{ci} = \frac{f'_{cc}}{\varepsilon_{cc}}$ $\quad \varepsilon_{cc} = \varepsilon_{co}\left[1+32.8(k_e\omega_w)^{1.9}\right]$ $k_e = \left(1-\frac{\Sigma w_i^2}{6b_c d_c}\right)\left(1-\frac{s}{2b_c}\right)\left(1-\frac{s}{2d_c}\right)$ $\quad E_c = 22{,}000\left(\frac{f'_c}{10}\right)^{0.3}$ $\omega_w = \frac{\rho_s f_{yh}}{f'_c}$ $\quad \varepsilon_{c50c} = \varepsilon_{co} + 0.091(k_e\omega_w)^{0.8}$ $\quad f'_{co} = 0.85 f'_c$	The stress-strain curve model from Nagashima et al. [11]. f'_{cc}, ε_{cc}, ε_{c50c} derived from linear regression analysis of experimental data on 108 large-scale specimens with 50 ≤ f'_c ≤ 125 MPa, f_{yh} ranges from 340 to 1390 MPa
Faimun et al. [8]	$f_c = f'_{cc}\left[\frac{k(\varepsilon/\varepsilon_{cc})}{k-1+(\varepsilon/\varepsilon_{cc})^k}\right]$ $\quad \frac{f'_{cc}}{f'_{co}} = 1.0+1.077\left(\frac{\rho_s f_{yh}}{f'_{co}}\right)^{0.7}$ $f_c = f'_{cc}\exp\left[k_1(\varepsilon-\varepsilon_{cc})^{k_2}\right]$ $\quad \varepsilon_{cc} = \varepsilon_{co} + 0.042\left(\frac{\rho_s f_{yh}}{f'_{co}}\right)^{1.7}$ $\varepsilon_{co} = \frac{f'_{co}}{E_c}\frac{n}{n-1}$ $\quad n = 0.8+\frac{f'_{co}}{17}$ $\quad \varepsilon_{c50c} = \varepsilon_{c50u} + 0.053\left(\frac{\rho_s f_{yh}}{f'_{co}}\right)^{1.1}$ $k_1 = \frac{\ln 0.5}{(\varepsilon_{c50c}-\varepsilon_{cc})^{k_2}}$ $\quad k = \frac{E_c}{E_c - \left(\frac{f'_{cc}}{\varepsilon_{cc}}\right)}$ $\quad \varepsilon_{c50u} = 0.004$ $k_2 = 0.58+4.22\left(\frac{\rho_s f_{yh}}{f'_{co}}\right)^{1.4}$ $\quad E_c = 3320\sqrt{f'_c}+6900$	f'_{cc}, ε_{cc}, ε_{c50c} are the modification of those by Cusson and Paultre [12] calculated by an iterative procedure for various f'_c, f_{yh}, and longitudinal and lateral steel configurations; $f'_{co} = 0.85 f'_c$

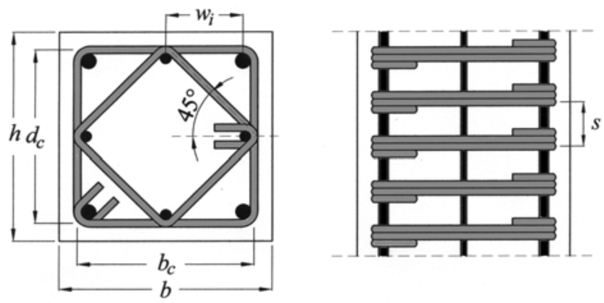


Fig. 2. Parameters used in the proposed model

$$\hat{y}_2 = 0.94 - 12.1x_2 + 164.68x_2^2 - 142.28\,(x_2 - 0.07)_+^2 \quad (6)$$

in which, $(x_2 - 0.07)_+^2 = (x_2 - 0.07)^2$, if $x_2 \geq 0.07$
$(x_2 - 0.07)_+^2 = 0$, if $x_2 < 0.07$

where $\hat{y}_2 = (\varepsilon_{cc}/\varepsilon_{co}) - 1$, and $x_2 = K_e\omega_w$. ε_{co} is the strain at the maximum stress of unconfined concrete, as given in Ref. [14]:

$$\varepsilon_{co} = \frac{0.70(f_c')^{0.31}}{1000} \quad (7)$$

where $\omega_w = \frac{\rho_s f_{yh}}{f_c'}$ is the mechanical ratio of transverse reinforcement.

The slope of the descending branch, which is the key parameter influencing the ductility, is determined by the strain when the maximum stress of confined concrete drops to 50 percent. The relationship between the ductility gain ($\varepsilon_{c50c} - \varepsilon_{co}$) and the effective capacity of transverse reinforcement $K_e\omega_w$ is given by Eq. (8).

$$\hat{y}_3 = -0.005 + 0.26x_3 - 0.66x_3^2 + 0.71\,(x_3 - 0.16)_+^2 \quad (8)$$

in which, $(x_3 - 0.16)_+^2 = (x_3 - 0.16)^2$, if $x_3 \geq 0.16$
$(x_3 - 0.16)_+^2 = 0$, if $x_3 < 0.16$

where $\hat{y}_3 = \varepsilon_{c50c} - \varepsilon_{co}$, and $x_3 = K_e\omega_w$.

Evaluation Of Confinement Models

The reviewed stress-strain models for confined normal- and high-strength concrete were applied to predict the results of experimental tests on large-scale specimens carried out and reported by Sheikh and Uzumeri (SU) [16]; Scott, Park, and Priestley (SPP) [17]; Nagashima, Sugano, Kimura, and Ichikawa (NSKI) [11]; Nishiyama, Fukushima, Watanabe, and Muguruma (NFWM) [18]; Cusson and Paultre (CP) [19]; and Razvi and Saatcioglu (RS) [20,21]. The experimental values of the concrete cylinder strength (f_c') reported in 100 × 200 mm size specimens were adjusted to the corresponding to a standard 150 × 300 mm cylinder by applying a conversion factor of 0.95 to account for the specimen size effect [16].

Table 2 presents the summary of the specimens considered and Fig. 3 gives the tie configurations in these specimens.

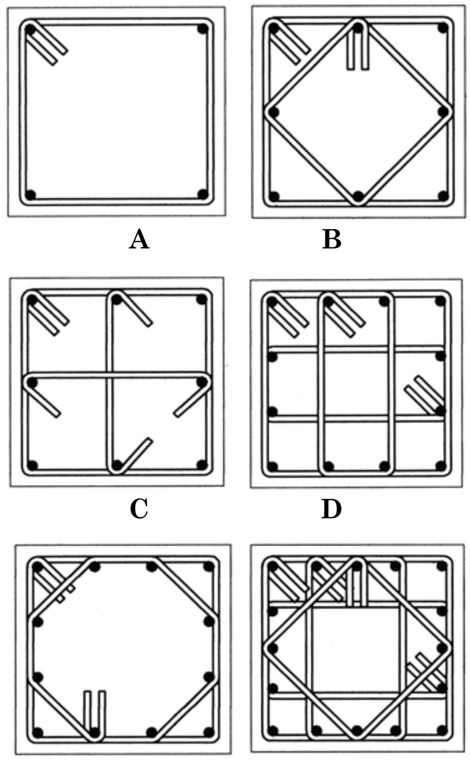


Fig. 3. Tie configuration of specimens

The reliability evaluation of the analytical models was based on the computation of the statistics of three key parameters. These are (see Fig. 1): i) the maximum confined concrete strength, f_{cc}'; ii) the confined concrete strain at the maximum strength, ε_{cc}; and iii) the confined concrete strain when the stress drops to 50 percent of the maximum confined concrete strength, ε_{c50c}. The strain at $0.5\,f_{cc}'$ is usually close to the point of failure due to hoop fracture and/or shear failure of the confined core [7].

Table 3 illustrates the statistics obtained for the three parameters for each analytical model alongside the number of specimens considered each time. It can be seen that the parameter for ductility (ε_{c50c}) was evaluated on the basis of fewer specimens, which is due to one of the following reasons: i) it was not possible to obtain the experimental values of the parameters from the reported data; ii) the analytical values of the parameters could not be deduced, because the descending branch of the analytical curve was parallel to the horizontal axis (Fafitis and Shah [6] model).

Table 2. Summary of the experimental data considered

No	Researchers	Details of the Specimens									
		Specimen Notation	Section mm	b_c mm	f'_c MPa	ρ_g %	f_y MPa	ρ_s %	s mm	f_{yh} MPa	Config.*
1	Susson and Paultre [19]	1A	235x235	195	95.4	2.20	406	2.90	50	410	A
2		2A			96.4			2.01	50	392	
3		3A			98.1			1.45	100	410	
4		4A			93.1	3.60	420	2.90	50		
5		5A			99.9			2.90	50	705	
6	Razvi and Saatcioglu [20]	CS-1	250x250	218.7	124	1.29	450	3.33	55	400	
7		CS-11			81			4.59	40		
8		CS-12						3.33	55		
9	Sheikh and Uzumeri [16]	2A1-1	305x305	267	36.5	1.72	372	0.80	57	478	B
10		2A1H-2			37			0.80	57	258	
11		4A3-7			40.9	3.33	385	1.66	76	508	
12		4A4-8			40.8			1.59	29	535	
13		4A5-9			40.5			2.39	76	365	
14		4A6-10			40.7			2.32	35	460	
15		4A1-13			31.3		439	0.80	57	535	
16		2A5-14			31.5	1.72	403	2.39	76	448	
17		2A6-15			31.7			2.32	35	478	
18	Scott et al. [17]	6	450x450	410	25.3	1.79	394	1.74	72	309	
19		7						1.74	72		
20		17			24.8			1.34	98		
21		18						1.74	72		
22		19						2.13	88	296	
23		20						2.93	64		
24	Nagashima et al. [11]	HH13LD	225x225	200	112.1	2.40	378	2.68	25	1386	
25		LL08LD			58.5			2.68	25	806	
26	Cusson and Paultre [19]	1B	235x235	195	95.4	2.20	450	3.43	50	392	
27		2B			96.4			2.25	50	414	
28		3B			98.1			2.48	100	410	
29		4B			93.1	3.60	406,450	3.43	50	392	
30		5B			99.9			3.43	50	77-	
31		6B			115.9		482,436	4.96	50	715	
32		7B			75.9			4.96	50		
33		8B			52.6			4.96	50		
34	Razvi and Saatcioglu [20]	CS-2	250x250	223.5	124	2.57	450	1.62	55	570	C
35		CS-4		222.5				2.17	55	1000	
36		CS-6		223.5				1.10	85	400	
37		CS-8		218.7				3.24	85		
38		CS-13		223.5	92			1.62	55	570	
39		CS-15		222.5	81			2.17	55	1000	
40		CS-17		223.5				1.10	85	400	
41		CS-19		218.7	92			3.24	85		
42		CS-22		222.5	60			1.40	85	1000	
43		CS-24		218.7				3.24	85	400	
44	Sheikh and Uzumeri [16]	4B3-19	305x305	267	33.4	3.66	392	1.80	100	460	
45		4B4-10			34.7			1.70	38	520	
46		4B6-21			35.5			2.40	48	478	
47	Nagashima et al. [11]	HH08LA	225x225	200	110.4	2.40	378	1.66	55	1386	
48		HH10LA						2.03	45		
49		HH13LA						2.61	35		
50		HH15LA						3.09	45	1366	
51		HH20LA			112.1			3.98	35		
52		HL06LA						2.03	45	806	
53		HL08LA						2.61	35		
54		LL05LA			57.3			1.66	55		
55		LL08LA						2.61	35		
56		LH08LA						1.66	55	1386	
57		LH13LA						2.61	35		
58		HH13MA			112.1		596	2.61	35		
59		HH13HA					803	2.61	35		
60		LL08MA			57.3		596	2.61	35	806	
61		LL08HA					803	2.61	35		
62		LH15LA			58.5		378	3.09	45	1366	
63		HH13MSA			112.1		596	2.61	35	1386	
64		HH13HSA					803	2.61	35		
65		LL08MSA			58.5		596	2.61	35	806	
66		LL08HSA					803	2.61	35		

* *see Fig. 3*

Table 2. Summary of the experimental data considered (cont.)

No	Researchers	Details of the Specimens									
		Specimen Notation	Section mm	b_c mm	f'_c MPa	ρ_g %	f_y MPa	ρ_s %	s mm	f_{yh} MPa	Config.*
67		1						3.98	31		
68		2						3.98	31		
69		3						3.98	31		
70		4		214	108.7				45	813	
71		5							60		
72		6							60		
73	Nishiyama et al. [18]	7	250x250			2.55	351		60		
74		8		216					31	840	
75		9							31		
76		10							31		
77		11		214	113.2				45	462	
78		12							60		
79		13							60		
80		14		216					31	481	
81		1D			100.4			4.80	50	392	
82		2D			96.4	2.20	450	3.08	50	414	
83		3D			98.1			3.39	100	410	D
84	Cusson and Paultre [19]	4D	235x235	195	93.1		406,450	4.09	50	392	
85		5D			99.9			4.69	50	770	
86		6D			113.6	3.60		4.69	50		
87		7D			67.9		482,467	4.69	50	680	
88		8D			55.6			4.69	50		
89		CS-3		223.5				2.52	55	570	
90		CS-5		222.5	124			1.55	120	1000	
91		CS-7		223.5				1.16	120	400	
92		CS-9		218.7				3.56	120		
93		CS-14		223.5	92			2.52	55	570	
94	Razvi and Saatcioglu [20]	CS-16	250x250	222.5	81	3.86	450	2.18	85	1000	
95		CS-18		223.5				1.73	85	400	
96		CS-20		218.7	92			5.04	85		
97		CS-23		222.5				1.55	120	1000	
98		CS-25		218.7	60			3.57	120	400	
99		CS-26		223.5				2.52	55	570	
100	Sheikh and Uzumeri [16]	4D3-22			35.5			1.60	82	460	
101		4D4-23	305X305	267	35.9	3.66	392	1.70	29	520	
102		4D6-24						2.30	38	478	
103		2			25.3			1.82	72		
104		3						1.82	72	309	
105		12					4.34	1.40	98		
106		13			24.8			1.82	72		
107		14						2.24	88	296	
108	Scott et al. [17]	15	450x450	410		1.86		3.09	64		
109		22						1.40	98		E
110		23			24.2		272	1.82	72	309	
111		24						2.24	88		
112		25						3.09	64		
113	Nagashima et al. [11]	HH13LB	225X225	200	118	2.40	378	2.60	27	1387	
114		LL08LB			62			2.60	27	807	
115		1C			95.4			3.62	50	392	
116	Cusson and Paultre [19]	2C			96.4	2.20	450	2.38	50	414	
117		3C	235X235	195	98.1			2.62	100	410	
118		4C			93.1	3.60	406,450	3.62	50	392	
119		5C			99.9			3.62	50	770	
120		4C1-3			36.4			0.76	50	558	
121		4C1H-4			36.7		372	0.76	50	285	
122		4C6-5			35	3.44		2.27	38	478	
123	Sheikh and Uzumeri [19]	4C6H-6			34.3			2.27	38	258	
124		4C3-11	305x305	267	40.7		409	1.62	95	460	F
125		4C4-12			40.8			1.52	25	720	
126		2C1-16			32.5			0.76	50	670	
127		2C5-17			32.9	2.22	414	2.37	100	460	
128		2C6-18			33.1			2.27	38	535	

** see Fig. 3.*

Table 3. Statistics of the ratio of experimental to analytical values for three key Parameters

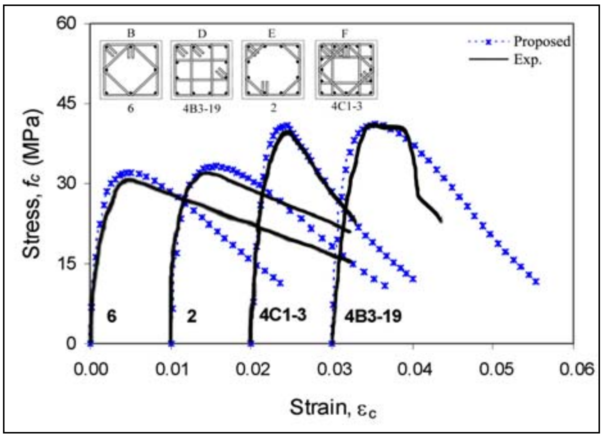
Model	Statistics	f'_{cc}	ε_{cc}	ε_{c50c}
(1)	(2)	(3)	(4)	(5)
Fafitis and Shah [6]	Mean	0.97 (128)	1.25 (109)	0.80 (73)
	St.Dev.	0.12 (128)	0.65 (109)	0.61 (73)
	COV (%)	12.7 (128)	51.9 (109)	76.2 (73)
Kappos and Konstantinidis [7]	Mean	0.96 (128)	1.35 (109)	0.90 (73)
	St.Dev.	0.08 (128)	0.70 (109)	0.35 (73)
	COV (%)	8.3 (128)	52.1 (109)	39.2 (73)
Faimun el al. [8]	Mean	0.99 (128)	0.99 (109)	1.16 (73)
	St.Dev.	0.13 (128)	0.50 (109)	0.41 (73)
	COV (%)	12.6 (128)	50.3 (109)	35.1 (73)
Proposed	Mean	1.02 (128)	0.90 (109)	0.96 (73)
	St.Dev.	0.09 (128)	0.48 (109)	0.36 (73)
	COV (%)	8.5 (128)	53.8 (109)	37.6 (73)

St. Dev.: Standard Deviation. COV: Coefficient of Variation.
For Columns (3), (4), and (5), the number in the parenthesis indicates the number of the specimens available or within the range of model applicability.

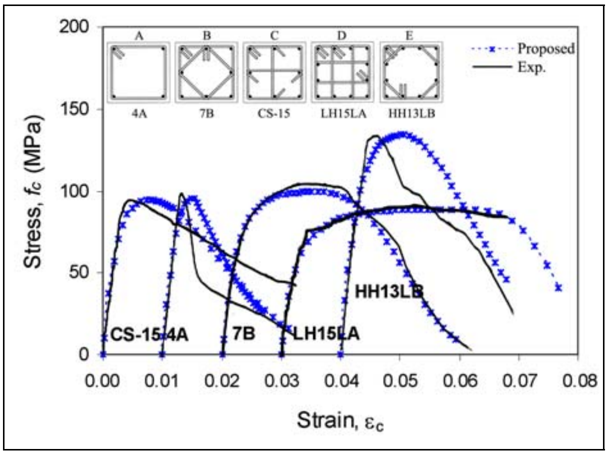
Selected comparisons of analytical and experimental stress-strain curves for four normal- and five high-strength concrete specimens with different tie configurations are given in Fig. 3. It can be seen from Fig. 4 that the overall proposed stress-strain curves give good prediction on the post-peak range (ε_{c50c}), the maximum confined concrete strength (f'_{cc}), and the strain at the maximum stress (ε_{cc}) either for confined normal- or high-strength concrete with various configuration of ties.

Fig. 5 shows the selected comparisons of proposed and existing analytical stress-strain curves suggested by other researchers [6,7,8] for nine specimens with various tie configurations. To show the accuracy of each model including the proposed model, the experimental stress-strain curves are also presented in the same figures. From the figures, it can be seen that there is a considerable discrepancy on the post-peak response of the stress-strain curves compared with the experimental results, particularly for Fafitis and Shah [6] model. Overall, the proposed model shows the best agreement with the experimental curves compared to the other existing models [6,7,8] either for normal- or well as high-strength concrete with various tie configurations.

The uncertainty related to the predicted confined concrete strength was found to be relatively low for all models, the coefficient of variation ranging from 8.3 to 12.7 percent (Table 3). This should be attributed to the fact that this particular parameter is straightforwardly defined. However, the equation proposed by Fafitis and Shah [6] in the mid 80's overestimates the strength increase of confined concrete in several cases, particularly for specimens, which combine high transverse volumetric ratio and high yield steel. The overall coefficient of variation is the highest among the models considered, that is 12.7 percent (Table 3). The proposed equations by Kappos and Konstantinidis [7] derived from linear statistical analysis overestimate the strength gain of confined normal-strength concrete specimens, as seen in Fig. 5.



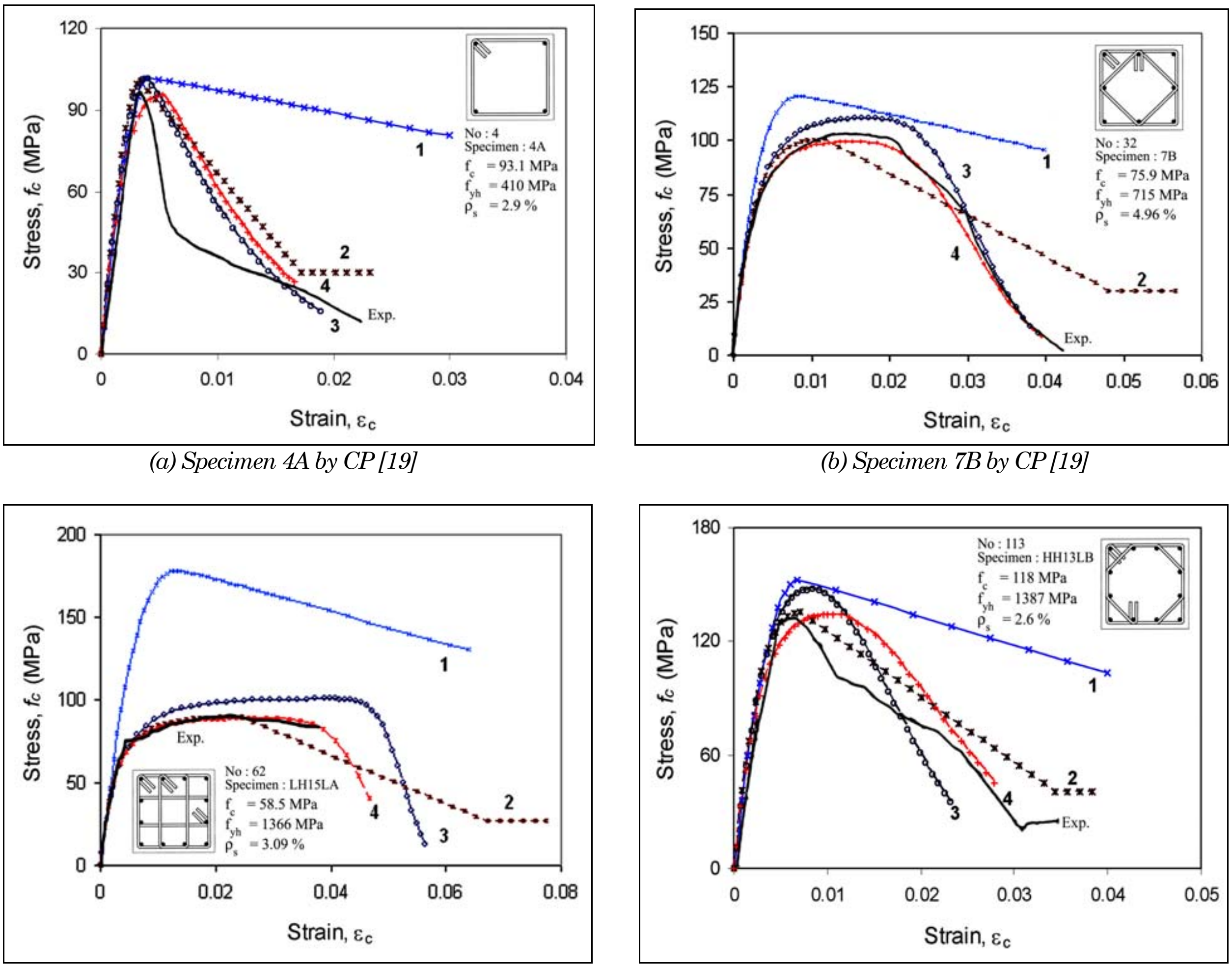
(a) Normal-strength concrete



(b) High-strength concrete

Fig. 4. Selected comparisons of proposed and experimental stress-strain curves of confined normal- and high-strength concrete

The coefficient of variation is 8.3 percent. Despite the lower coefficient of variation given by Kappos and Konstantinidis [7] model, significant deviations in the strength gain have been found in the confined normal-strength concrete specimens as shown in Fig. 5. Thus, the model is not applicable for use in normal-strength concrete. The proposed equations by Faimun et al. [8] resulted in a more accurate prediction on the strength gain of confined normal-strength concrete. However, it slightly overestimates the strength increase of confined high-strength concrete. The coefficient of variation is 12.6 percent. Scatter diagrams for confined concrete strength (f'_{cc}) are shown in Fig. 6.



(a) Specimen 4A by CP [19]

(b) Specimen 7B by CP [19]

(c) Specimen LH15LA by NSKI [11]

(d) Specimen HH13LB by NSKI [11]

Legends:
1. *Fafitis and Shah [6]*
2. *Kappos and Konstantinidis [7]*
3. *Faimun, Aji, Tavio, and Suprobo [8]*
4. *Proposed*

Notes:
CP: Cusson and Paultre
NSKI: Nagashima, Sugano, Kimura, and Ichikawa

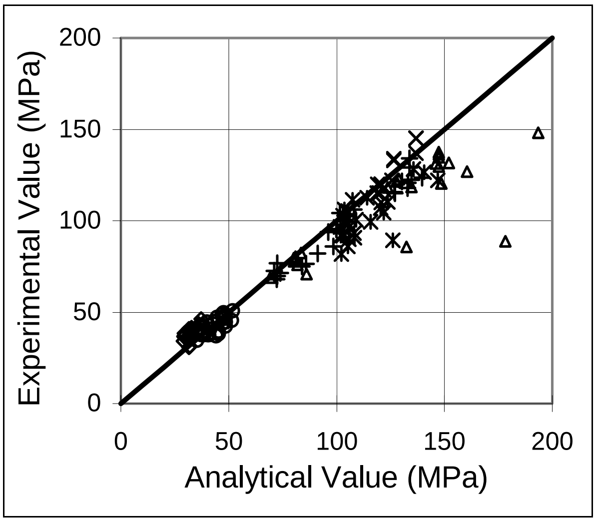
Fig. 5. Selected comparisons of proposed and existing analytical stress-strain curves

High scatter, in the range of 50.3 to 53.8 percent, was found for the second selected key parameter, which involves the strain at peak stress of confined concrete (ε_{cc}), as can be seen in Fig. 7. Measurement of strain is subject to a number of ambiguities, mainly because it depends on the length along which the 'average' strain is calculated, i.e. on the location of the LVDT's along the specimen.

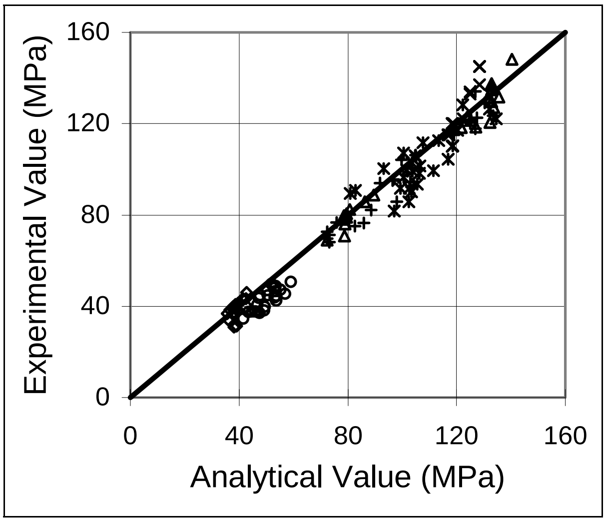
Fig. 8 shows the scatter diagrams for the ultimate concrete strain along the descending branch when the stress drops to 50 percent of the maximum strength of confined concrete (ε_{50c}). The uncertainty in the prediction of ε_{50c} was found to be relatively high. The coefficient of variation of ε_{50c} ranges from 35 to 76 percent, on the basis of 73 specimens. Fafitis and Shah [6] model overestimates ε_{50c} by 76 percent.

It is obvious that the lack of experimental results on realistic size columns at the time these models were developed led to a rather poor description of the falling branch. On the other hand, the coefficients of variation given by Kappos and Konstantinidis [7] as well as Faimun et al. [8] are 39 and 35 percent, respectively, on the basis of 73 specimens.

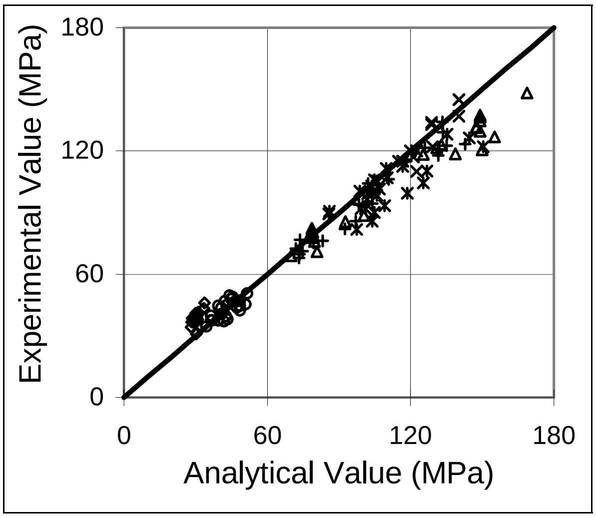
Although, the statistical evaluation of all the key parameters indicates that the proposed model provides comparable accuracy to the other models considered in the present study, the proposed stress-strain curves show higher consistency and better agreement with the experimental curves, as compared to the other models considered in the paper.



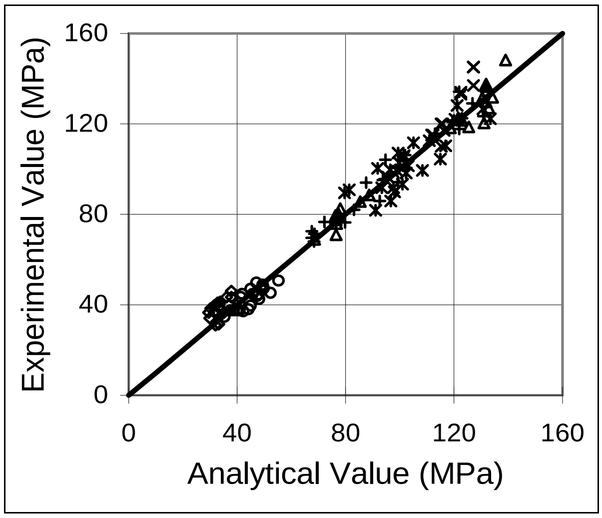
(a) FS [6]



(b) KK [7]



(c) FATS [8]

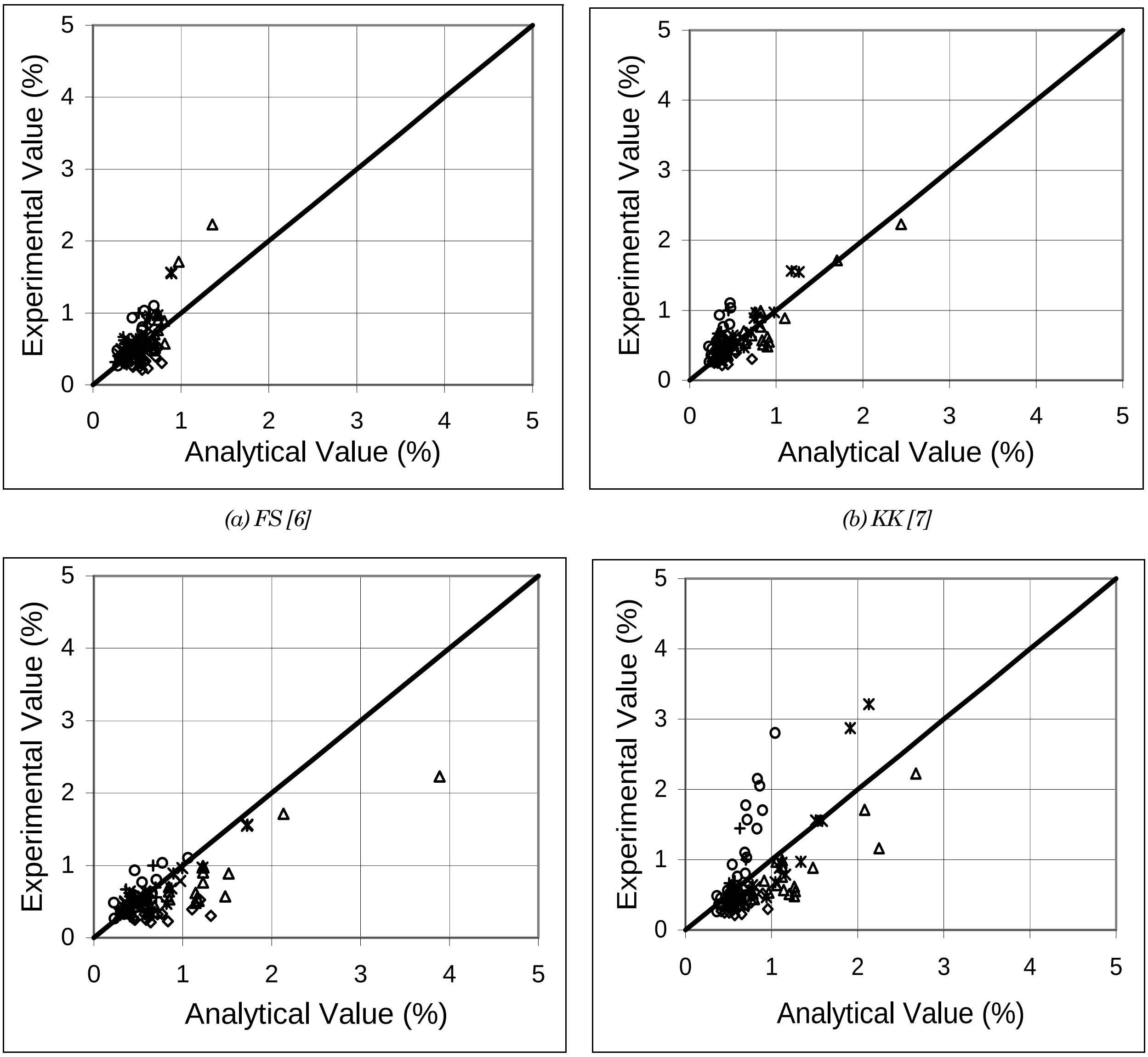


(d) Proposed

Notes:
FS: Fafitis and Shah
KK: Kappos and Konstantinidis
FATS: Faimun, Aji, Tavio, and Suprobo

Legends:
o *Sheikh and Uzumeri [16]*
◊ *Scott, Park, and Priestley [17]*
Δ *Nishiyama, Fukushima, Watanabe, and Muguruma [18]*
× *Nagashima, Sugano, Kimura, and Ichikawa [11]*
∗ *Cusson and Paultre [19]*
+ *Razvi and Saatcioglu [20]*

Fig. 6. Scatter diagrams for confined concrete strength (f'_{cc})



(a) FS [6]

(b) KK [7]

(c) FATS [8]

(d) Proposed

Notes:
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Legends:
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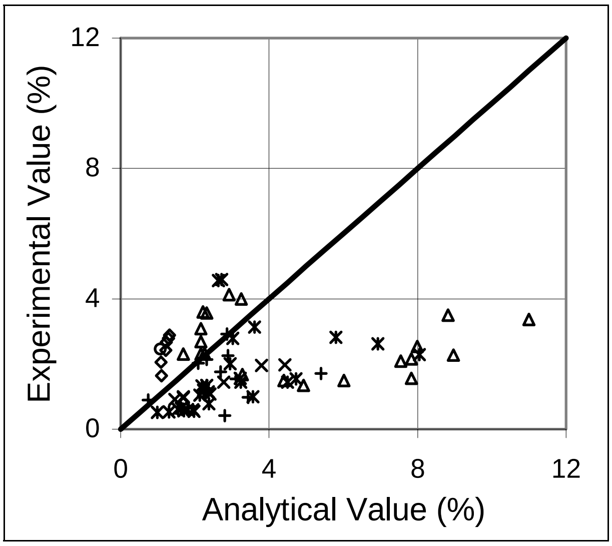
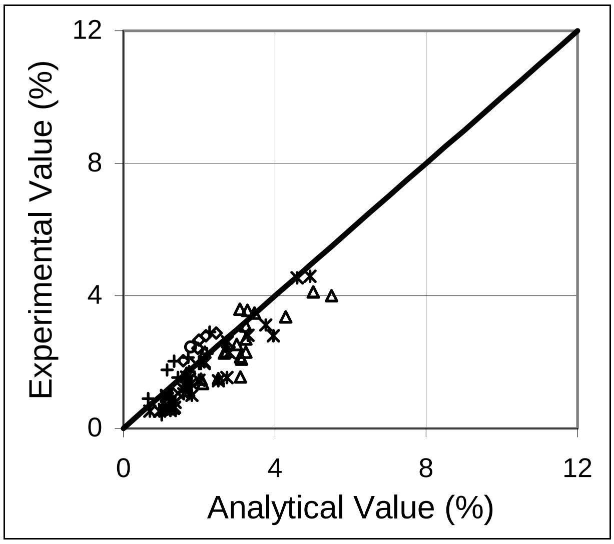
∗ *Cusson and Paultre [19]*
+ *Razvi and Saatcioglu [20]*

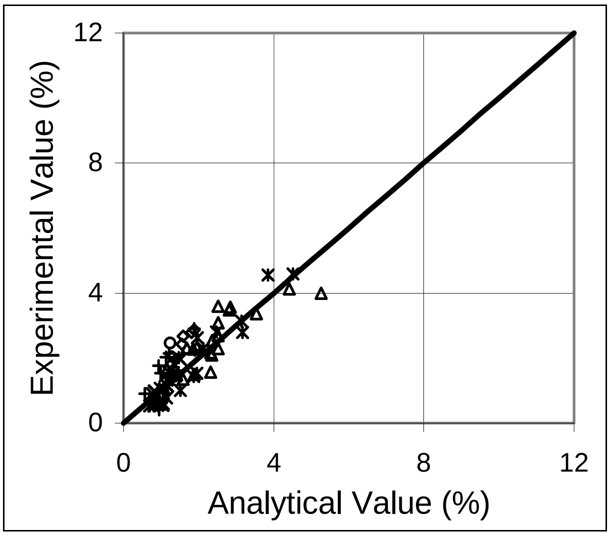
Fig. 7. Scatter diagrams for strain at peak stress of confined concrete (ε_{cc})



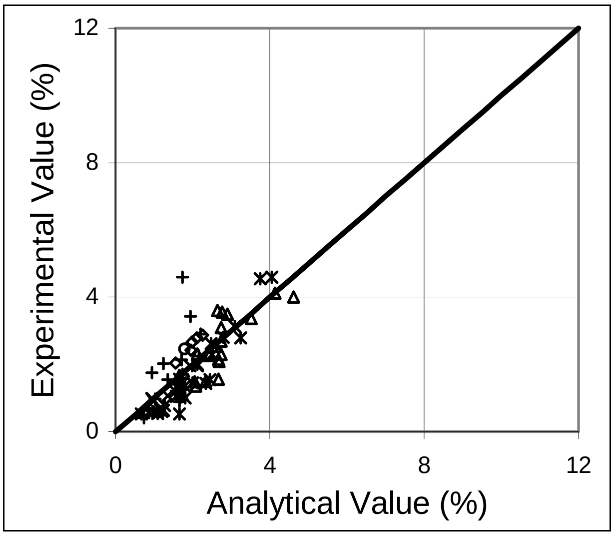
(a) FS [6]



(b) KK [7]



(c) FATS [8]



(d) Proposed

Notes:
FS: Fafitis and Shah
KK: Kappos and Konstantinidis
FATS: Faimun, Aji, Tavio, and Suprobo

Legends:
o *Sheikh and Uzumeri [16]*
◊ *Scott, Park, and Priestley [17]*
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∗ *Cusson and Paultre [19]*
+ *Razvi and Saatcioglu [20]*

Fig. 8. Scatter diagrams for strain when the stress drops to 50 percent of the maximum confined concrete strength (ε_{50c})

Conclusions

Three existing analytical models for concrete confined by rectilinear hoops/ties were studied, and statistical evaluation of three key parameters of the stress-strain curve was presented. Then, using as the basis the existing model that provided the best curve fitting, a new statistical model was developed, using a spline nonparametric regression analysis involving a total of 128 test data. The following conclusions can be drawn from this study.

1. All models predict the ascending branch of the stress-strain curve fairly well, whereas the predicted descending branch is not consistent.
2. Analytical models derived based on experimental data from normal- or high-strength concrete, and/or small-scale lightly confined columns were shown to have the highest uncertainties, as anticipated.
3. The maximum strength of confined concrete is well predicted by all models. Higher uncertainties were found for ductility (ε_{c50c}).
4. The statistical model proposed in the present study using the spline nonparametric regression analysis, which is based on a large experimental database, resulted in fairly low uncertainties equivalent to the other models considered.
5. The proposed stress-strain curve closely predicts the experimental results of confined normal- as well as high-strength concrete with normal- or high strength steel ties.

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