

# Evaluation of Crack Initiation Points of Load-Carrying Cruciform Welded Joints Based on Effective Notch Stress Approach

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**Abstract:** For fatigue assessment, recently effective notch stress (ENS) approach is accepted as a valid alternative to fracture mechanics approaches in Japan. However, at least one study showed that the fatigue assessment at weld root of load-carrying cruciform welded joints based on ENS approach does not capture the influence of plate thickness, weld size and weld penetration. Consequently, there are still uncertainties about the fatigue assessment. This paper discusses the evaluation method of the joints based on ENS approach. Effects of weld size, weld penetration, and plate thickness on ENS are determined by using fine-meshed FEM analyses. Validity of this method is determined by comparing the evaluation results with fracture mechanics approach. It is shown that results from this method are comparable to those of the previous study and the results seem to be conservative when ratio of weld penetration to plate thickness becomes smaller and plate thickness becomes larger.

**Keywords:** Crack initiation point; effective notch stress; load-carrying cruciform welded joints; weld root.

## Introduction

Steel frame piers are commonly used for highway viaducts in urban areas in Japan [1]. It is observed that fatigue cracks occurred in beam-to-column connections of these frame piers [2]. Causes of fatigue cracks of beam-to-column connections were experimentally investigated in previous studies [3-7]. Miki et al. [3] concluded that one of the causes is incomplete penetration of welded joints. This may occur from poor welding practice in constructions, inspite of full penetration weld specified in the design [6]. Therefore, in order to ensure good performance and safety of structures, retrofiting works, such as installing bolt splices and drilling holes to remove incomplete penetration and fatigue cracks, have been conducted for severely damaged beam-to-column connections [1,4,7].

However, it is not feasible to conduct these retrofit works to slightly damaged beam-to-column connections, because the cost is high and the construction time is long. Therefore, various retrofiting strategies, which are more efficient, have been proposed [4,8,9] for slightly damaged beam-to-column connections.

One of the efficient retrofit works, which satisfies the requirements for slightly damaged beam-to-column connections, is weld repair with special care for post-welding cracking and deformation. Recently, Miki et al. [8] proposed weld repair using low temperature transformation welding material which introduces beneficial compressive residual stress at treated weld toe and reduces post-weld deformation. This method is cost efficient and is easy for inspections since weld repair can change fatigue crack initiation points from embedded incomplete penetration to weld toes on structure surfaces.

Despite the development of those retrofit works, there are still uncertainties about the evaluation method of fatigue crack initiation points. Kainuma et al. [10] and Kainuma and Mori [11] investigated fatigue crack initiation points of load-carrying fillet welded cruciform joints, as shown in Figure 1, using fatigue crack propagation analysis. They proposed the following equations to evaluate fatigue crack initiation points of the joints [10].

$$\text{Root failure: } S/t \leq \alpha \quad (1)$$

$$\text{Toe failure: } S/t \geq \alpha \quad (2)$$

$$\alpha = -1.83 \left( \frac{p_w}{t} \right) + 1.20 \quad (3)$$

where  $S$  is the weld size,  $p_w$  is weld penetration, and  $t$  is plate thickness

Although welding joints of beam-to-column connections may be categorized as load-carrying fillet welded cruciform joints, in reality this connection has complex three-dimensional incomplete penetration and also its shape and size depend on assembly of the connection.

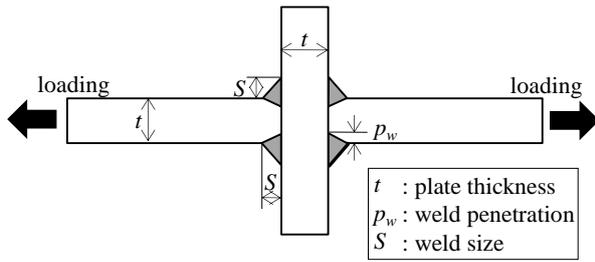
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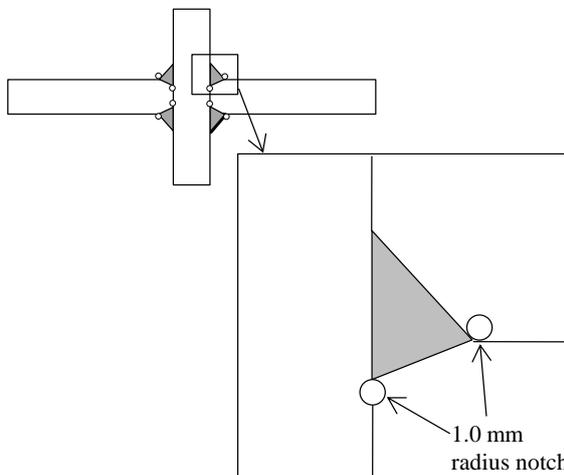
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**Figure 1.** Load-Carrying Cruciform Welded Joints

It is therefore believed that results based on small-scale welded joint fatigue tests without complex three-dimensional incomplete penetrations may not be able to capture the actual fatigue crack initiation points of beam-to-column connections.

Recently, effective notch stress (ENS) approach has been included in the International Institute of Welding (IIW) fatigue design recommendations [12]. Effective notches usually are illustrated as shown in Figure 2. Effective notches, which have 1 mm radius notches, when the plate thickness is greater than or equal to 5 mm, can add notch stresses induced by weld profiles to structural stresses. As shown in Figure 2, this approach is applicable to both weld toes and weld roots. This approach is recently accepted as a valid alternative to fracture mechanics approaches in Japan [7,13,14,15]. Park and Miki [14] recently investigated the fatigue assessment of welded joints based on the ENS approach using previous large-scale joint specimen test results, and concluded that it is possible to distinguish whether fatigue cracks will initiate at the weld toe or the weld root with three-dimensional incomplete penetrations by using the ENS approach. This study shows a possibility of further extension of evaluation of fatigue crack initiation points of beam-to-column connections with complex three-dimensional incomplete penetrations based on the ENS approach.



**Figure 2.** Effective Notches at Weld Toes and Weld Root

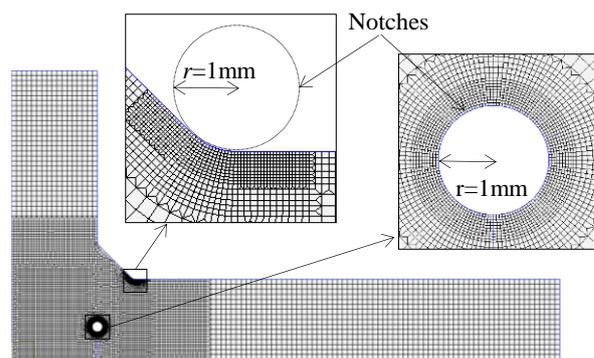
However, Mori and Myoken [15] carried out fatigue strength evaluation at weld root by using the ENS approach based on small-scale welded joint fatigue tests and concluded that the evaluation over estimates the influence of thickness and under estimates influence of weld size and weld penetration. Therefore, the validity of the evaluation of fatigue crack initiation points on load-carrying cruciform welded joints even without complex three-dimensional incomplete penetrations still remains unclear.

In this paper, results from fine-meshed FEM analyses using the ENS approach on load-carrying cruciform welded joints are presented. Effects of weld size, weld penetration, and plate thickness on effective notch stress are able to be determined. This study also check validity of evaluation of fatigue crack initiation points based on the ENS approach by comparing evaluation results from the ENS approach with fracture mechanics approach. The evaluation results from the fracture mechanics approach were obtained from previous study [10]. This paper adds analysis results and provides new conclusions based on the authors' previous studies [16-18].

### Analytical Model

Figure 1 shows the analysis model. Uniform tensile loading was applied to the main plates. The main plate and cross plate thicknesses were kept the same. The root gap was assumed to be 0 mm. Geometric parameters, such as weld size  $S$ , weld penetration  $p_w$ , and plate thickness  $t$ , are shown in Figure 1. From previous studies, fatigue strength is influenced by these parameters [10]. Therefore, this paper focuses on these parameters. Various  $S$ ,  $p_w$ , and  $t$  were investigated as listed in Table 1.

A sample FEM model is shown in Figure 3, which was made in finite element analysis software DIANA [19].



**Figure 3.** FEM Model

**Table 1.** Analysis Cases

$t$ (mm)	$S$ (mm)	$S/t$	$p_w/t$	$t$ (mm)	$S$ (mm)	$S/t$	$p_w/t$
9	7.0	0.8	0.00	34	7.0	0.2	0.00
			0.25				0.15
			0.27				0.25
			0.29				0.30
			0.30				0.35
	9.0	1.0	0.00				0.40
			0.20				0.42
			0.21				0.44
			0.25				0.44
			0.25				0.46
17	7.0	0.4	0.00	17.0	0.5	0.00	
			0.15			0.25	
			0.25			0.35	
			0.30			0.40	
			0.35			0.41	
			0.38			0.43	
			0.40			0.00	
			0.41			0.30	
			0.41			0.00	
			0.43			0.30	
	17.0	1.0	0.00	34.0	1.0	0.00	
			0.15	40.8	1.2	0.30	
			0.25	51.0	1.5	0.00	
			0.27	57.8	1.7	0.20	
			0.28	61.2	1.8	0.00	
			0.30	0.00			
			0.30	0.25			
			0.30	0.30			
			0.30	0.45			
			0.30	0.46			
20.4	1.2	0.00	75	7.0	0.1	0.00	
		0.00				0.25	
		0.20				0.30	
		0.00				0.45	
		0.00				0.46	
		0.00				0.47	
		0.00				0.50	
		0.00				0.00	
		0.00				0.25	
		0.00				0.43	
25.5	1.5	0.00	17.0	0.2	0.45		
		0.00			0.45		
		0.00			0.50		
27.2	1.6	0.00	0.00				
		0.00	0.00				

Plane strain condition was used, because usually the width of welded joints is sufficient to be assumed as the plane strain condition. FEM models used are quarter-scale models. Quadrilateral elements with 4 nodes were mainly used to the models. Minimum element size is 0.05 mm around notches. Young's modulus  $E$  and Poisson's ratio  $\nu$  are set to be 200 kN/mm<sup>2</sup> and 0.3, respectively.

Elastic FEM analyses were conducted. ENS is the local maximum principal stress at notches which is directly obtained by using imaginary notches to weld root and weld toe. A one mm radius notch was recommended in IIW fatigue design recommendations [12]. Park and Miki [14] also showed that crack initiation points are able to be determined by using 1.0 mm notch radius. Thus, 1.0 mm notch radius was used in this study as shown in Figure 3.

## Results of Evaluation

### Evaluation Method of Crack Initiation Points

As mentioned before, the previous study [10] carried out fatigue crack propagation analysis using fracture mechanics approach in order to evaluate fatigue crack initiation points. The study carried out the analysis for both weld toe and weld root, and compared both results to distinguish whether fatigue cracks will initiate from weld root or weld toe. The fatigue crack growth rate was assumed as follows,

$$\frac{da}{dN} = C(\Delta K^m - \Delta K_{th}^m) \quad (4)$$

where  $a$  is crack length,  $N$  is number of cycles, and  $\Delta K$  is stress intensity factor range. The material constants,  $C$  and  $m$ , and threshold stress intensity factor range,  $\Delta K_{th}$ , are assumed as  $1.5 \times 10^{-11}$ , 2.75, and 2.9 MPa m<sup>1/2</sup>, respectively, which were determined based on the fatigue design recommendation of Japan Society of Steel Construction [20]. For weld toes, the general form of the following stress intensity factor range equation proposed by Albrecht and Yamada [21] was used.

$$\Delta K = F_S \cdot F_T \cdot F_E \cdot F_G \cdot \Delta \sigma \sqrt{\pi a} \quad (5)$$

where  $\Delta \sigma$  is the stress range, and  $F_S$ ,  $F_T$ ,  $F_E$ , and  $F_G$  are the correction factors. On the other hand, for the calculation of  $\Delta K$  at weld root, the following equations proposed by Frank and Fisher [22] were used.

$$\Delta K = \frac{\Delta \sigma \left\{ A_1 + A_2 \left( \frac{a}{w} \right) \right\} \sqrt{\pi a \sec \left( \frac{\pi a}{2w} \right)}}{1 + \frac{2S}{t}} \quad (6)$$

$$A_1 = 0.528 + 3.287 \left( \frac{S}{t} \right) - 4.361 \left( \frac{S}{t} \right)^2 + 3.696 \left( \frac{S}{t} \right)^3 - 1.875 \left( \frac{S}{t} \right)^4 + 0.415 \left( \frac{S}{t} \right)^5 \quad (7)$$

$$A_2 = 0.218 + 2.717 \left( \frac{S}{t} \right) - 10.171 \left( \frac{S}{t} \right)^2 + 13.122 \left( \frac{S}{t} \right)^3 - 7.755 \left( \frac{S}{t} \right)^4 + 1.783 \left( \frac{S}{t} \right)^5 \quad (8)$$

where  
 $w = S + t/2$ ,  $0 \leq w/a \leq 0.7$ ,  $0.2 \leq S/t \leq 1.2$ .

Applicability of the above equations were validated by Mori et al. [23]. Based on the comparison of both crack propagation analysis results, Kainuma et al. [10] defined a critical weld size  $S_L$ . As weld size increases, weld toe becomes the fatigue crack initiation point since the weld size is more than the critical weld size  $S_L$ . In the previous study [10], the critical weld size was normalized by plate thickness  $t$  as  $S/t$  and the relationship between  $S/t$  and  $p_w/t$

was shown. Therefore, to compare with the previous study [10], the relationship between  $S_L/t$  and  $p_w/t$  are the main focus in this study.

Figure 4 shows a comparison of ENS at weld root and weld toe, and shows the maximum principal stress distribution. It can be seen that high stress occurred at weld toe and weld root. Maximum stresses at around weld toe,  $\sigma_{toe}$ , and at around weld root,  $\sigma_{root}$ , are used to ENS. By comparing both ENS, the point, where higher ENS occurred, is expected to be a crack initiation point. In case of Figure 4,  $\sigma_{root}$  is higher than  $\sigma_{toe}$ , so weld root is expected to be a crack initiation point.

Figure 5 shows examples of the relationship between ENS and  $S/t$  when  $t$  is 17 mm and 34 mm while  $p_w/t$  is 0. It can be seen that  $\sigma_{toe}$  and  $\sigma_{root}$  decrease with the increase in  $S/t$ , and eventually,  $\sigma_{root}$  becomes lower than  $\sigma_{toe}$  as shown in Figure 5. Therefore, it can be obtained that  $S_L/t$  when  $t$  is 17 mm is about 1.5 and when  $t$  is 34 mm is about 1.8.

### Comparison of Results

Figure 6 shows comparison of the relationship between  $S_L/t$  and  $p_w/t$  obtained from this and the previous studies. It shows that the results, when  $t$  is taken as 9 mm and when  $p_w/t$  is large, are comparable to those of the previous study. Therefore, it may be concluded that the results are independent on  $t$  when  $p_w/t$  is large. On the other hand, the results using ENS approach do not agree well with the previous study when  $p_w/t$  is small and  $t$  is large. This overestimation increases with the decrease of  $p_w/t$  and increase of  $t$ . More specifically, when  $p_w/t$  is kept constant as 0,  $S_L/t$  when  $t$  is 17 mm is about 1.5 and when  $t$  is 34 mm is about 1.8. Thus, it can be concluded that the results are dependent on  $t$  when  $p_w/t$  is smaller, and therefore they seem to be conservative when  $p_w/t$  is smaller and  $t$  is larger.

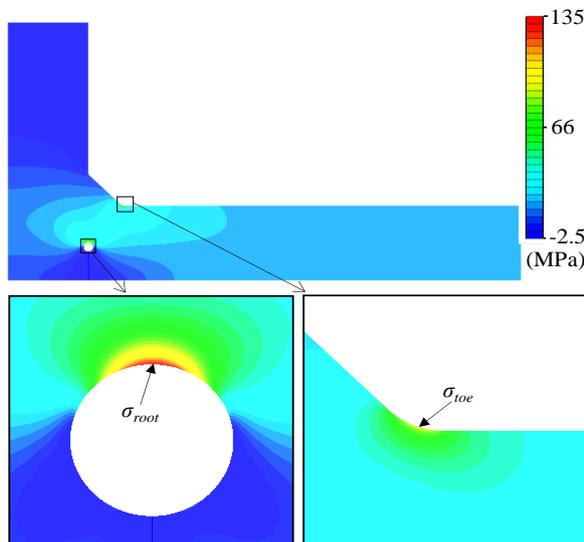
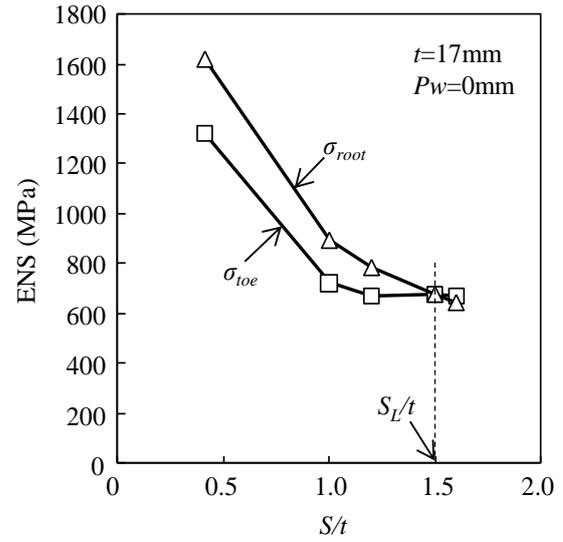
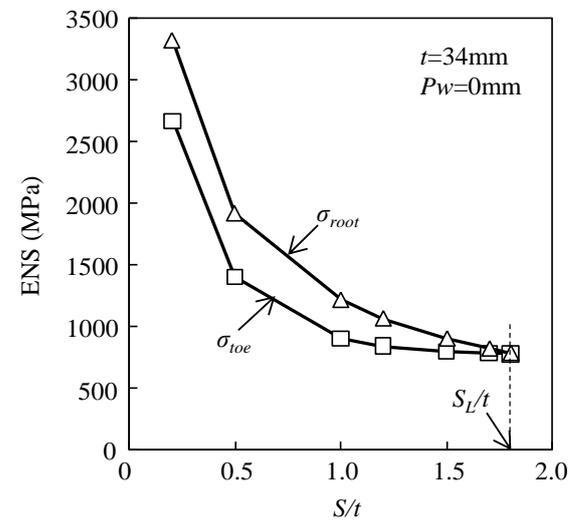


Figure 4. Comparison of ENS at Weld Root and Weld Toe



(a)  $t=17\text{mm}$

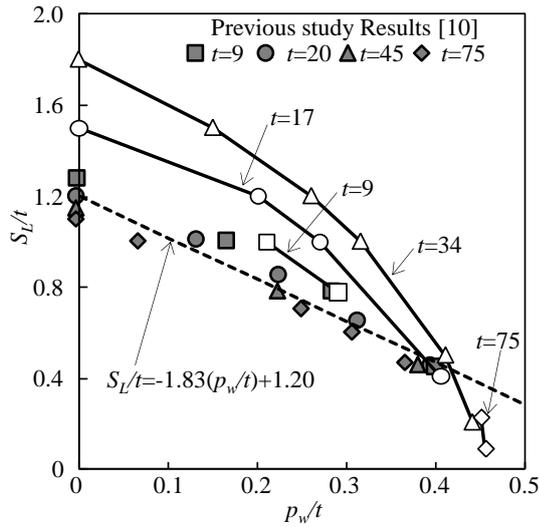


(b)  $t=34\text{mm}$

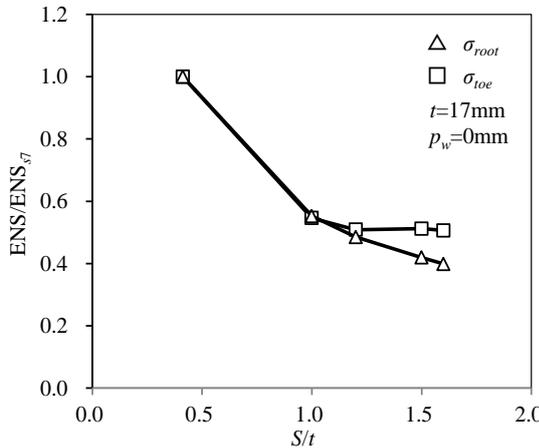
Figure 5. Relationship between ENS and  $S/t$

### Influences of Parameters

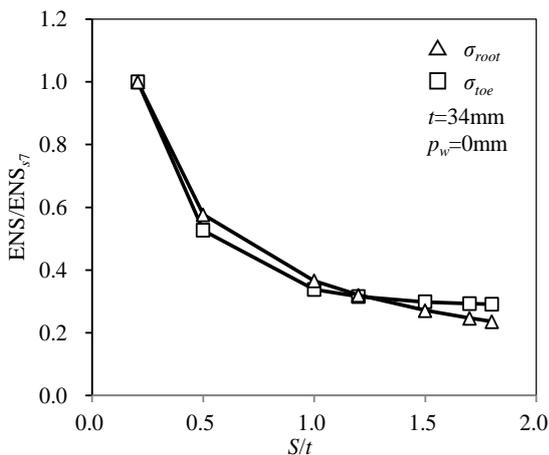
As a follow up, influences of  $S$ ,  $p_w$ , and  $t$  were investigated individually. Figure 7 shows the relationship between  $S/t$  and ENS (normalized by  $ENS_{S7}$ , i.e., when  $S$  is 7 mm) when  $t$  is 17 mm and 34 mm while  $p_w$  is 0. From Figure 7, it can be observed that  $\sigma_{root}$  and  $\sigma_{toe}$  are almost the same when  $S/t$  is small. Both  $\sigma_{root}$  and  $\sigma_{toe}$  decrease significantly with the increase of  $S/t$  when  $S/t$  is less than 1.0. This suggests that a retrofit work using weld repair with special care for post-welding cracking and deformation is effective when  $S/t$  is smaller than 1.0. On the other hand, although both  $\sigma_{root}$  and  $\sigma_{toe}$  decrease with the increase of  $S/t$  when  $S/t$  is more than 1.0, decrease of  $\sigma_{root}$  is greater than that of  $\sigma_{toe}$  when  $t$  is 17 mm. However, the difference between  $\sigma_{root}$  and  $\sigma_{toe}$  is not significant when  $t$  is 34 mm.



**Figure 6.** Comparison of Relationship between  $S_L/t$  and  $p_w/t$  obtained from Present and the Previous Studies [10]



(a)  $t=17\text{mm}$



(b)  $t=34\text{mm}$

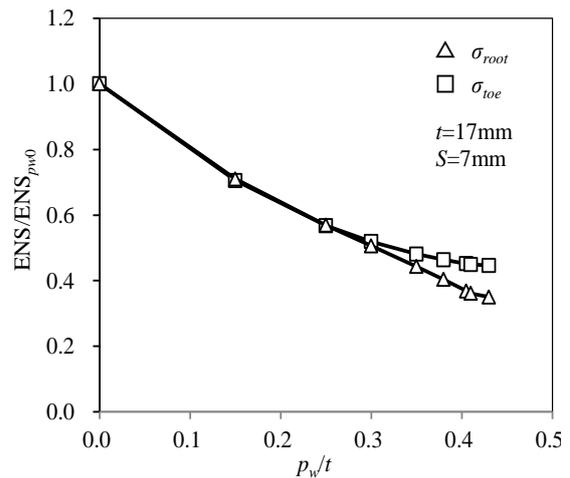
**Figure 7.** Relationship between  $S/t$  and  $ENS/ENS_{s7}$

Figure 8 shows the relationship between  $p_w/t$  and ENS (normalized by  $ENS_{p_w0}$ , i.e., when  $p_w$  is 0 mm) when  $t$  is 17 mm and 34 mm while  $S$  is 7 mm. It can be observed that the tendency of decrease of both  $\sigma_{root}$

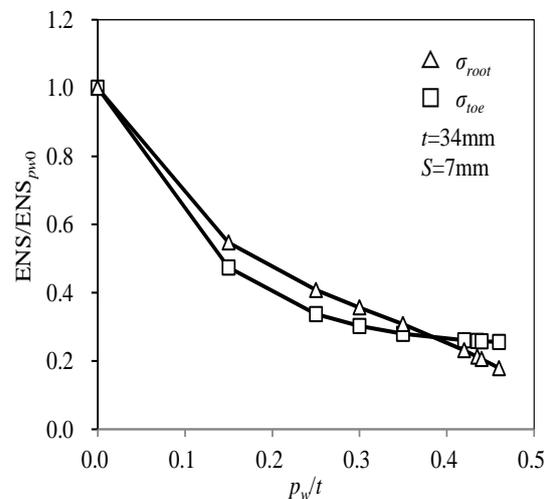
and  $\sigma_{toe}$  is similar to Figure 7. In addition, although the decrease of  $\sigma_{root}$  is greater than that of  $\sigma_{toe}$  when  $t$  is 17 mm, the difference between  $\sigma_{root}$  and  $\sigma_{toe}$  is not significant when  $t$  is 34 mm.

Figure 9 shows the relationship between  $t$  and ENS (normalized by  $ENS_{t9}$ , i.e., when  $t$  is 9 mm) when  $p_w/t$  is 0.25 and  $p_w/t$  is 0 while  $S$  is 7 mm. It can be observed that the increase of  $\sigma_{root}$  due to an increase of  $t$  is larger than that of  $\sigma_{toe}$ . The difference between  $\sigma_{root}$  and  $\sigma_{toe}$  is more significant when  $p_w/t$  is 0.25. Thus, the influence of  $t$  on  $\sigma_{root}$  is larger than that on  $\sigma_{toe}$ .

Therefore, it can be concluded that the most likely reason why the evaluation results using ENS approach seems to be conservative when  $p_w/t$  becomes smaller and  $t$  becomes larger is that influences of  $S$  and  $p_w$  decrease with the increase of  $t$  and influence of  $t$  on  $\sigma_{root}$  remain larger compared to  $\sigma_{toe}$  even though  $t$  becomes larger.



(a)  $t=17\text{mm}$



(b)  $t=34\text{mm}$

**Figure 8.** Relationship between  $p_w/t$  and  $ENS/ENS_{p_w0}$

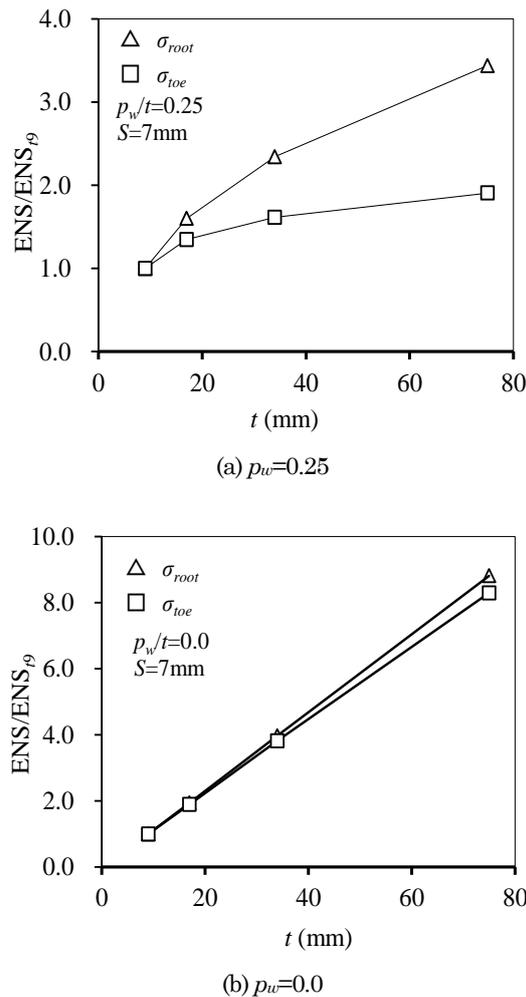


Figure 9. Relationship between  $t$  and  $ENS/ENS_{9}$

## Conclusions

An evaluation method of fatigue crack initiation points using the effective notch stress (ENS) approach used to load-carrying cruciform welded joints has been discussed in this paper. Validity of the evaluation method was checked by comparing with previous study [10]. Although the analysis cases were limited, it is shown from the study that:

1. The results of the evaluation method are comparable to those of the previous study [10] when the ratio of the weld penetration  $p_w$  to the plate thickness  $t$ ,  $p_w/t$ , is large or  $t$  is small, but the results seem to be conservative when  $p_w/t$  is smaller and  $t$  is larger.
2. The most likely explanation for the conservative results is that the influences of weld size  $S$  and  $p_w$  decrease with the increase of  $t$ , and influences of  $t$  on maximum stresses at around weld root  $\sigma_{root}$  remain larger than that on maximum stresses at around weld toe  $\sigma_{toe}$  even though  $t$  becomes larger.
3. Influence of  $S$  on weld root when the ratio of weld size  $S$  to plate thickness  $t$ ,  $S/t$ , is smaller than 1.0 is significant. Thus, it is concluded that a retrofit

work using weld repair with special care for post-welding cracking and deformation is effective when  $S/t$  is smaller than 1.0.

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