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A Hybrid AHP-QRVE Model for Consistency Verification in Property Valuation

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Abstract

The conventional Quantitative Rating Value Estimation (QRVE) method for building valuation often introduces bias because it relies heavily on subjective Likert-scale ratings. This study aims to address the inherent weaknesses of Likert scales used in the QRVE method by mitigating the threshold ambiguity and appraiser cognitive inconsistencies. This study proposes a hybrid methodology by integrating the pairwise comparison matrix from the Analytic Hierarchy Process (AHP) into the QRVE framework. The mathematical foundation of AHP transforms absolute assessments into relative ones, with the Consistency Ratio (CR) serving as a verification metric through the induced matrix $C = AA - nA$ (n represents the total number of evaluated properties (including the subject property). The analysis results show that the AHP-QRVE integration successfully detects valuation errors and inconsistencies between comparable objects when applying conventional Likert scales. Through numerical simulations, this method has provided more objective assessment outputs, even greater audit transparency for the Public Appraisal Services Office.

INTRODUCTION

Building valuation is a complex process involving various qualitative and quantitative attributes that must be converted into accurate monetary values [1]. In the construction industry, the conversion of these two attribute types is crucial to the accuracy of this valuation, which ultimately leads to a contractor's decision to submit a bid [2]. To bridge the complexity of these attributes in real practice, one of the techniques commonly used in Property Valuation in Indonesia is the sales comparison approach through the Quantitative Rating Value Estimation (QRVE) method, using a Likert scale to assign scores to variables such as a building's physical condition, location, and utility [3]. However, this approach suffers from a significant research gap: it basically depends on the subjective judgment of appraisers, leading to threshold ambiguity and cognitive inconsistencies. Because conventional QRVE lacks a built-in mathematical verification mechanism to check the consistency of these judgments, it frequently introduces human bias, which ultimately compromises the accuracy and reliability of the final property valuation.

Based on these problems, this research seeks a solution to minimize assessor bias and inconsistency in the QRVE method. Therefore, this research aims to develop a hybrid method by incorporating the pairwise comparison system from AHP into QRVE. Through this AHP approach, assessments that were previously subjectively estimated are transformed to more measurable (relative) comparisons, using the induced matrix $\boldsymbol{C} = \boldsymbol{AA} - n\boldsymbol{A}$ as an internal

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consistency-verification mechanism. Practically, this framework provides a more robust, objective, and auditable tool for property appraisers and Public Appraisal Services Offices, thereby increasing the transparency and stability of building appraisal results.

The rest of our paper is organized as follows. In the next section, we will provide some literature reviews regarding the use of pairwise comparison and AHP in property (real estate) valuation. We will then state several important theorems to form a solid mathematical foundation for our proposal, i.e., the equivalence of Likert-like numbers and a perfectly consistent matrix, a simple method that we can use to easily check the consistency of a pairwise matrix as well as our proposed methodology. We will also illustrate with examples all of the theorems as well as the benefit of our proposed methodology, together with some discussion. Finally, we provide our conclusion and further research.

LITERATURE REVIEW

Multi-Criteria Decision Analysis (MCDA) and the QRVE Framework

The use of Multi-Criteria Decision Analysis (MCDA) in property valuation has been extensively studied over the years, with most MCDA-based methods relying on pairwise comparison techniques. For example, Cañas et al. [4] applied the MACBETH method [5] to aggregate multi-criteria opinions in residential rental valuation using anchor points and linear interpolation. However, this approach has a fundamental weakness as it relies heavily on extreme values (highest and lowest) as its primary reference. On the other hand, Ginevičius et al. [6] demonstrated the Simple Additive Weighting (SAW) method for office rental ranking. Although SAW is an attractive MCDA approach, it is essentially identical to the traditional Quantitative Rating Value Estimation (QRVE) technique. Consequently, both SAW and conventional QRVE suffer from the same problem, which leads to appraiser subjectivity bias due to the lack of internal consistency control mechanisms [3].

In the standard Quantitative Rating Value Estimation (QRVE) procedure, the assessment process is generally carried out through five main stages, which are:

1) Decide on a set of m characteristics (x_i) that, the appraiser believes, can influence the market value (of the subject as well as its comparables), e.g., location, accessibility, design, and other relevant attributes. Assign weights to each one of them (w_i) $\forall i = 1, \dots, m$
2) Assign scores (ordinal/Likert-type scale, usually between 1 – 5) to the subject of evaluation as well as its comparables
3) Calculate the quality rating/score for the subject as well as its comparables $qs_j = \sum_{i=1}^{m} x_i w_i$
4) Run a simple linear regression in the form $v_j = b_0 + b_1(qs_j) + \varepsilon_j$ to obtain b_0 and b_1 using the c comparables
5) Use the value of b_0 and b_1 together with the quality score of the subject, *i.e.*, qs_0 to obtain the estimated value of the subject $v_0 = b_0 + b_1(qs_0)$

Some consideration in the first stage about defining the basis of weight (w_i) assignments on those variables $\forall i = 1, \dots, m$ and the best weight to obtain the largest R-squared was obtained via Multiple Linear Regression [3]. However, when an appraiser assigns scores to the subject of evaluation using the conventional Likert scale (stage 2), it has fundamental weaknesses in terms of objectivity and internal consistency [7,8]. The use of absolute Likert scales often fails to capture the nuances of differences between comparable buildings. In addition to the consistency issue, the Likert Scale has a threshold of ambiguity, where the boundary between one level of quality and another is subjective and not operationally defined, resulting in data with high variance across appraisers [9,10]. Without validation tools, appraisers may assign inconsistent scores, ultimately distorting the accuracy of the final valuation. Therefore, a shift from absolute scoring to a structured relative assessment mechanism is required to resolve these cognitive limitations, establishing a clear logical foundation for incorporating the Analytic Hierarchy Process (AHP).

AHP in Property Valuation

To address this limitation, this study proposes a transformation from absolute (Likert) assessments to relative assessments through pairwise comparison, part of the Analytic Hierarchy Process (AHP), as an alternative to stage 2. The use of AHP is already implemented widely, such as for business and industries, but also in the medical area when they evaluate studies related to the Likert Scale [11-14].

Unlike the absolute and independent Likert scale, the AHP forces assessors to make pairwise comparisons (relative assessments), the consistency of which can be mathematically measured using the Consistency Ratio (CR). By integrating the mathematical foundations of the AHP into traditional assessment systems, assessment inconsistencies can be systematically reduced. Table 1 illustrates the problem with the direct assignment of scores in stage 2. Consider a simple characteristic (e.g., x_1= Location) for 15 different comparables ($P_{01} - P_{15}$) and a subject of evaluation (P_{00}).

Table 1. Example of Scores (1-5) Assigned to the Location Characteristic

Project	Location	Project	Location
P01	4	P09	5
P02	5	P10	4
P03	5	P11	5
P04	5	P12	5
P05	4	P13	5
P06	4	P14	5
P07	5	P15	5
P08	5	P00	5

When an appraiser assigns scores as in Table 1, there are no checks and balances – the appraiser can make a mistake in the assignment of the scores, and nobody may have noticed it. Furthermore, the theory of decision making explains that it is difficult for humans to make decisions when they face many options (i.e., comparables), as mentioned in the Paradox of Choice [15] and Predictably Irrational [16]. Lastly, it should be noted that assigning a value to the subject of evaluation, P_{00} = 5, does not affect the model. The model is only constructed using the 15 data points of comparable buildings. Then, the subject of evaluation is being evaluated. Notice the example above that all values are only 4 and 5. These numbers are very subjective, and there is no way to know whether they are correct. More importantly, there is no checking for its consistency. This is the main problem that we try to address in this paper – a simple scientific alternative method using Analytic Hierarchy Process (AHP) that can be used to provide the appraiser with some checks and balances even if (s)he uses a subjective approach. Furthermore, given that the subject of evaluation (P_{00}) is also included in the comparison from the very beginning, when the appraiser does not carefully pick the comparison, the predicted value of P_{00} may not make sense, and it can alert the appraiser when something is not very accurate.

Unel & Yarpir [16] as well as Ozsubasi & Ertas [17] showed how to use the Analytic Hierarchy Process (AHP) in valuation. Both papers used AHP for valuation in different regions of Turkey. It is important to explain here that both papers used AHP to obtain the weights of explanatory variables, *i.e.*, w_i. Furthermore, the paper by Unel & Yarpir provided a comparison between weights that are obtained *via* AHP *vs.* using Multiple Linear Regression (MLR). They used R-squared, MAPE, & RMSE to compare the results from AHP *vs.* MLR, and the AHP model seems to produce a better result. Most importantly, the use of AHP means that they relied on pairwise comparison. It is also important to explain that our usage and proposal of AHP usage here is different from both Unel & Yapir [16] as well as Ozsubasi & Ertas [17]. The very recent paper by Aihie *et.al.* [18] concluded that AHP can be used as a tool to support property appraisal. They provided a real example with a building in the Southeast of Calgary, Canada. The value obtained using AHP is comparable to the value that is obtained using the Direct Comparison Approach (DCA). This capability highlights how replacing absolute scaling with relative comparisons can enhance valuation reliability, setting the stage for a deeper analysis of pairwise versus Likert approaches.

Methodological Synthesis: Pairwise Comparison vs. Likert Scale in Practice

Regarding the comparison between the Likert scale and pairwise comparison, several studies by Phelps et al. [19], Hoeijmakers et al. [20], and Wang et al. [21] demonstrate that pairwise comparison provides significantly higher reliability. Although these studies originate from the healthcare sector—specifically in Computed Tomography (CT) scan evaluations- their methodological findings strongly support the fundamental premise of this study. Furthermore, recent advancements presented by the research group from Università Iuav in Venezia highlight the expanding implementation of AHP in real estate valuation [22], which was later extended into non-linear optimization frameworks [23]. While the core objectives of this study align closely with the work of Copiello et al. [24], substantial differences remain. This study provides a more robust mathematical bridge connecting AHP-like mechanisms with traditional valuation methods that rely on Likert-scale inputs. The primary distinction lies in the customization of the consistency check. This proposed approach does not restrict the pairwise comparison to have a scale between 1 and 9 (Saaty standard [22]) but, like in AHP (can be adapted to any number – in practice, at several valuation companies in Indonesia, the typical scale is 1 – 5). Although previous studies have demonstrated success in applying AHP and pairwise comparison methods, most of these studies are still limited to determining variable weights in general or applied outside the context of traditional QRVE assessment. To clarify the theoretical position and practical contributions of this study, Table 2 summarizes the differences between these previous studies and the new approach we propose in this study.

Table 2. Summary Comparison of Previous Studies and the Proposed Approach

Research Focus / Authors	Method & Validation Mechanism (VM) Used	Study Context	Research Gap addressed / Novelty of This Study
Unel & Yalpir [16]; Özsubaşı & Ertaş [17]	AHP vs. Multiple Linear Regression (MLR) VM: Standard AHP Consistency Check	Property Valuation (Weight determination)	Focuses on comparing AHP weights against MLR but does not address the subjective scoring issues inside the QRVE framework.
Phelps et al. [19]; Hoeijmakers et al. [20]; Wang [21]	Pairwise Comparison vs. Likert Scale VM: Varies	Healthcare Industry (CT Scan evaluations)	Proves the superior reliability of pairwise comparison over Likert scales, but their findings are outside the property appraisal domain.
Bonifaci et al. [23]; Copiello et al. [24]	AHP and Non-Linear Optimization VM: Mathematical Optimization / Presentation at ERES	Real Estate Valuation	Implements AHP in valuation but relies on rigid frameworks or complex optimization rather than linking it back to traditional Likert-type inputs.
This Study (Proposed Approach)	Hybrid AHP-QRVE VM: Internal Consistency Verification through $C = AA - nA$	Building and Property Valuation	Directly links AHP-like pairwise logic with traditional Likert inputs. Crucially, it removes the rigid 1–9 AHP scale restriction, adapting it to a flexible scale (e.g., 1–5) to match the practical standard of valuation companies in Indonesia.

MATHEMATICAL FOUNDATION OF THE PROPOSED METHODOLOGY

Given that humans tend to face challenges when there are multiple alternatives at once, and the resemblance of our problems to the MCDA problem, we decided to adopt an MCDA method that utilizes pairwise comparison, i.e., and we adopt an AHP-like method. There are two other essential reasons for proposing the role of AHP in valuation: the pairwise comparison (AHP-like) is still relatively easy to implement, and it provides a check of consistency. The checking of the consistency of AHP has been a subject of numerous research studies since AHP was first introduced by Saaty [22]. Saaty suggested the use of a scale between 1 and 9, but we can easily change the number to 1 – 5 or to any number. We first establish the equivalency of AHP with the traditional approach using a 1 – 5 (or 1 – 9 scale) by recognizing Theorem 1 below [25] for a very basic explanation from the AHP type of approach, as well as Furtado & Johnson [26-27] for a more rigorous explanation. Then we perform simple consistency checks that can be incorporated into our proposed algorithm, and we establish our methodology for strengthening the subjective valuation from the appraiser. We adapted our methodology to the practice of QRVE, which is very common in Indonesia.

A. Theorem 1:

A pairwise comparison matrix $\boldsymbol{A} = \left[a_{ij}\right]_{n\times n}$ is perfectly consistent if and only if it corresponds to a simple weight vector $\boldsymbol{w}$. To avoid ambiguity, let $\boldsymbol{c}$ denote the number of comparable properties, $n = \boldsymbol{c} + 1$ denote the total number of properties (including the single subject property), and $\boldsymbol{m}$ denote the number of characteristics.

Proof:
Straightforward from the fundamentals of the AHP equation: $\boldsymbol{Aw} - \boldsymbol{nw} = \boldsymbol{0}$ and the definition of perfectly consistent pairwise comparison matrix – see also the Introduction part of the paper by Furtado & Johnson [27]. To illustrate Theorem 1, consider the following simple Likert-type scale for 2 comparables and 1 subject for a particular criterion (*e.g.*, Location):

$$\boldsymbol{w}^T = [P_1 \quad P_2 \quad P_3] = [2 \quad 3 \quad 1]$$

The corresponding consistent pairwise comparison matrix will be given by the following matrix:

$$A = \begin{bmatrix} 1 & 0.67 & 2 \\ 1.50 & 1 & 3 \\ 0.50 & 0.33 & 1 \end{bmatrix}$$

Using the knowledge from AHP, we can easily verify that $\boldsymbol{Aw} = [6 \quad 9 \quad 3]^T = 3 \times [2 \quad 3 \quad 1]^T$. Notice that $n = 3 = |A|$. In addition, following the notation of Furtado & Johnson [29], we have: $\boldsymbol{w}^{-T} = [P_1 \quad P_2 \quad P_3] = [0.5 \quad 0.33 \quad 1]$ and as a result we can produce a pairwise consistent matrix using:

$$\boldsymbol{ww}^{-T} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} [0.5 \quad 0.33 \quad 1] = \begin{bmatrix} 1 & 0.67 & 2 \\ 1.50 & 1 & 3 \\ 0.50 & 0.33 & 1 \end{bmatrix} = \boldsymbol{A}$$

Furthermore, recently Ergu et. al. [28] provided a very important theorem for a perfectly consistent pairwise comparison matrix, shown in the theorem below.

B. Theorem 2:

A pairwise comparison matrix $A = [a_{ij}]_{n \times n}$ is perfectly consistent if and only if $\boldsymbol{C} = \boldsymbol{AA} - n\boldsymbol{A}$ should be a zero matrix (of size $n \times n$), i.e.,

$$\boldsymbol{C} = \boldsymbol{AA} - n\boldsymbol{A} = \boldsymbol{0} \quad (1)$$

Proof of Reverse Direction ($\boldsymbol{C}$ = 0 ==> Perfect Consistency):

Assume $\boldsymbol{C} = \boldsymbol{AA} - n\boldsymbol{A} = \boldsymbol{0}$, which implies $\boldsymbol{AA} = n\boldsymbol{A}$. Multiplying both sides by an $n\,x\,1$ vector of ones $e = [1,\ 1, \ldots, 1]^T$, we obtain $\boldsymbol{A}(\boldsymbol{Ae}) = n(\boldsymbol{Ae})$. Let $\boldsymbol{w} = \boldsymbol{Ae}$. Since $\boldsymbol{A}$ is a positive matrix, $\boldsymbol{w} > \boldsymbol{0}$, showing that n is an eigenvalue of $\boldsymbol{A}$ with corresponding positive eigenvector $\boldsymbol{w}$. According to Saaty's consistency theorem for pairwise comparison matrices, a positive reciprocal matrix $\boldsymbol{A}$ of order n has a maximum eigenvalue $\lambda_{max} = n$ if and only if $\boldsymbol{A}$ is perfectly consistent (i.e., $a_{ij} = w_i/w_j$, so $a_{ij}a_{jk} = a_{ik}$). Thus, $\boldsymbol{C}$ = 0 guarantees perfect consistency.

We also provide Lemma 1 from Agustin *et.al.* [4] as Theorem 3 below to relate it back to the QRVE.

C. Theorem 3 (Lemma 1 in Agustin et.al.[3]*):*

The most linear ("best" as defined by Zaric) quality score for comparable j (i.e., qs_j) can be obtained by defining the weight w_i of characteristic i as:

$$w_i = \frac{c_i}{\sum_{i=1}^{m} c_i} \quad \forall i = 1, \ldots, m \qquad \text{if all } c_i > 0 \quad (2)$$

where: $\boldsymbol{c}^T = [c_1 \quad \cdots \quad c_m]$ is the Ordinary Least Square (OLS) estimate for $\boldsymbol{v} = \boldsymbol{Xc} + \boldsymbol{u}$ with m variables. Now, we are in a position to state the new approach for the valuation process to replace step 2 (assigning Likert scale) above. To enhance the clarity of this new approach, a conceptual research framework of the proposed QRVE with AHP-style methodology is illustrated in Figure 1.

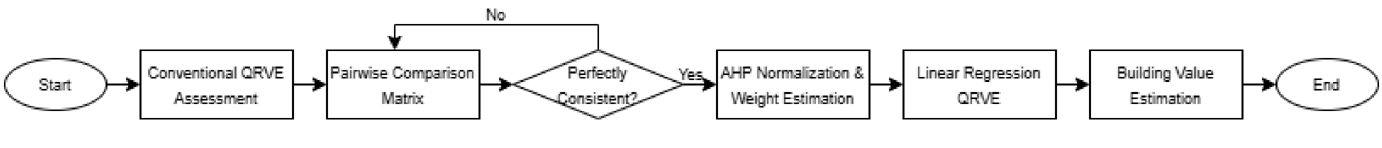

Figure 1. Conceptual Flowchart of the Proposed Hybrid AHP-QRVE Methodology

The process begins with the initial valuation inputs, which are then converted into a pairwise comparison matrix. A crucial quality-control stage is the consistency check; if the matrix fails to meet the consistency threshold, the appraiser is prompted to revise the inputs. Once consistency is validated, the workflow proceeds to AHP normalization and weight estimation, which subsequently feed into the linear regression analysis to produce the final building value estimation. Then, the new approach of the proposed QRVE with AHP-style methodology becomes as follows.

1) Decide on a set of m characteristics (x_i) that the *appraiser* believes can influence the market value (of the subject as well as its comparables), e.g., Location, Accessibility, Design, etc
2) Ask the *appraiser* for every characteristic (i.e., $\forall x_i\ i = 1, \ldots, m$) to perform a pairwise comparison among all comparables and the subject of evaluations $(n = c + 1)$ objects. Hence, we construct as many as m pairwise comparison matrices, each matrix has size $n\ x\ n$

3) Check the consistency of every pairwise comparison matrix $A_{n\times n}$ (i) $\forall i = 1, \dots, m$ (for every criterion) using Theorem 2 above. There are 2 possible outcomes:
   3.1) If the induced matrix $C_{n\times n} = \mathbf{0}$, then go to Step 4
   3.2) Else, use Step I – III to identify the location of inconsistent pairwise comparison (see Section 3.2 in Ergu et.al. [28]) together with some basic transitivity property, and ask the *appraiser* to make a correction and repeat Step 2 until the pairwise comparison matrix is perfectly consistent

It is important to emphasize that step 3 is the checking and balancing stage that is missing in the traditional use of the Likert scale. Of course, in practice, we can relax the perfectly consistent requirement of this step, and replace it with the Consistency Index proposed by Saaty, but it is recommended to tighten the consistency index to be between 2.5% – 5% (instead of the normal 10% as originally suggested by Saaty). However, we feel that for this application, it is better to have checks and balances so that the *appraiser* can make it consistent. Remember that our approach is not limiting the *appraiser* to just integer input with scale 1 – 9 (hence, another reason we called it AHP-like). Our proposed methodology is more focused on the pairwise comparison to produce a consistent transversal reciprocal matrix.

4) Performs the AHP process to get the normalized score for every characteristic for every perfectly (near-perfect) consistent pairwise comparison matrix
5) Once we have normalized values from all perfectly consistent pairwise comparison matrices, we can consider 2 situations:
   5.1) Apply Theorem 3 above (Lemma 1 from Agustin et.al. [3]) if the number of comparables is bigger than the number of characteristics, i.e., $c > m$, obtain the best weights for all characteristics, i.e., w_i $\forall i = 1, \cdots, m$ that maximized the R-squared
   5.2) Else (i.e., $c \le m$), apply the traditional AHP with pairwise comparison among all characteristics to decide the weight for each characteristics, i.e., w_i $\forall i = 1, \cdots, m$
6) Calculate the quality rating/score for the subject as well as its comparables $qs_j = \sum_{i=1}^{m} x_i w_i$
7) Run a simple linear regression in the form $v_j = b_0 + b_1(qs_j) + \varepsilon_j$ to obtain b_0 and b_1 using the c comparables
8) Use the value of b_0 and b_1 together with the quality score of the subject, i.e., qs_0 to obtain the estimated value of the subject $v_0 = b_0 + b_1(qs_0)$

RESULTS AND DISCUSSION

To illustrate the equivalence of consistent matrices and original Likert style, consider a rather simple example below of 5 projects × 3 criteria and a subject (Project 0) that we want to predict the fair market rent. We first establish the following corollary of Theorem 1 and illustrate with an example from data in Table 3.

Table 3. Likert Scale of Original Data with 3 Criteria

	Criterion 1	Criterion 2	Criterion 3	Rent (USD)
Project 1	2	3	2	200
Project 2	3	2	4	250
Project 3	4	2	3	350
Project 4	1	5	5	275
Project 5	5	3	1	300
Project 0	4	2	3	???

A solution to the Multiple Linear Regression model $\boldsymbol{Y} = \boldsymbol{Xb}$, *i.e.*, $\boldsymbol{b} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}$ will be scaled by the $\sum x_i$ when $\boldsymbol{X}$ is replaced by its eigenvector from perfect pairwise comparison matrix. The corresponding consistent matrices for Criteria 1, 2, and 3 for all projects (including the subject of Project 0) are given in Table 4.

Furthermore, the corresponding eigenvectors for the above 3 matrices are given in Table 5. For completeness, we also put the rent value (in USD).

Running a simple multiple linear regression on the data in Tables 3 and 5 produces the following equations (C1: Criterion 1, C2: Criterion 2, and C3: Criterion 3).

$$Rent = -17.33 + 49.72 \times C1 + 18.47 \times C2 + 29.26 \times C3 \tag{3a}$$

$$Rent = -17.33 + 944.60 \times Eigen\_C1 + 313.92 \times Eigen\_C2 + 526.70 \times Eigen\_C3 \tag{3b}$$

Table 4. Pairwise Comparison Matrix of Original Data with 3 Criteria

Criterion 1

	P1	**P2**	**P3**	**P4**	**P5**	**P0**
P1	1.000	0.667	0.500	2.000	0.400	0.500
P2	1.500	1.000	0.750	3.000	0.600	0.750
P3	2.000	1.333	1.000	4.000	0.800	1.000
P4	0.500	0.333	0.250	1.000	0.200	0.250
P5	2.500	1.667	1.250	5.000	1.000	1.250
P0	2.000	1.333	1.000	4.000	0.800	1.000

Criterion 2

	P1	**P2**	**P3**	**P4**	**P5**	**P0**
P1	1.000	1.500	1.500	0.600	1.000	1.500
P2	0.667	1.000	1.000	0.400	0.667	1.000
P3	0.667	1.000	1.000	0.400	0.667	1.000
P4	1.667	2.500	2.500	1.000	1.667	2.500
P5	1.000	1.500	1.500	0.600	1.000	1.500
P0	0.667	1.000	1.000	0.400	0.667	1.000

Criterion 3

	P1	**P2**	**P3**	**P4**	**P5**	**P0**
P1	1.000	0.500	0.667	0.400	2.000	0.667
P2	2.000	1.000	1.333	0.800	4.000	1.333
P3	1.500	0.750	1.000	0.600	3.000	1.000
P4	2.500	1.250	1.667	1.000	5.000	1.667
P5	0.500	0.250	0.333	0.200	1.000	0.333
P0	1.500	0.750	1.000	0.600	3.000	1.000

Table 5. Eigenvectors of Criteria 1 – 3 from Consistent Matrices Correspond to the Eigenvalue = 6

Eigenvector	**C1**	**C2**	**C3**	**Rent (USD)**
Project 1	0.1053	0.1765	0.1111	200
Project 2	0.1579	0.1176	0.2222	250
Project 3	0.2105	0.1176	0.1667	350
Project 4	0.0526	0.2941	0.2778	275
Project 5	0.2632	0.1765	0.0556	300
Project 0	0.2105	0.1176	0.1667	???

One can easily verify the relationship of coefficients in (3a) and (3b) that

$$\frac{944.60}{49.72} = (2 + 3 + 4 + 1 + 5 + 4) = 19,$$

$$\frac{313.92}{18.47} = (3 + 2 + 2 + 5 + 3 + 2) = 17,$$

and lastly

$$\frac{526.70}{29.26} = (2 + 4 + 3 + 5 + 1 + 3) = 18$$

as in Corollary 4. It should also be very easy to confirm that both equations (3a) and (3b) have the same R-squared and other statistical properties (F-statistics, t-Statistics, etc.). Lastly, estimated rent for P0 can be obtained from either equation to produce:

$$-17.33 + 49.72 \times 4 + 18.47 \times 2 + 29.26 \times 3 =$$
$$-17.33 + 944.60 \times 0.2105 + 313.92 \times 0.1176 + 526.70 \times 0.1667 = \$306.25.$$

In addition to the proof of Theorem 1, this example illustrates the equivalence of the Likert style and fully consistent pairwise comparison matrices. Now, we can illustrate this with the same example, but the difference in the process, i.e., the most important part of our proposal, is to have a check and balance step. Imagine that the appraiser made a small mistake in giving the rating of Project 0 (P0). Instead of $[4 \quad 2 \quad 3]$, (s)he entered: $[3 \quad 3 \quad 3]$ for 3 different criteria, then the value of the Rent would be predicted to become: $-17.33 + 49.72 \times 3 + 18.47 \times 3 + 29.26 \times 3 =$ \$275. There is no more checking. If we apply the pairwise comparison, we will ask the appraiser 5 times for each criterion, *i.e.*, how P0 compares to P1 (Project 1), P2, P3, P4, and P5 for each criterion. Let's suppose for Criterion 1, the appraiser makes the following comparison mistake – notice that (s)he only makes a very minor mistake in comparing P0 to P2. Instead of inputting the correct number as shown in Table 6(a), the appraiser input the incorrect number as depicted in Table 6(b).

Table 6(a). Example of Correct Input

Criterion 1	**P1**	**P2**	**P3**	**P4**	**P5**	**P0**
P0	2.000	1.333	1.000	4.000	0.800	1.000

Table 6(b). Example of Incorrect Input

Criterion 1	**P1**	**P2**	**P3**	**P4**	**P5**	**P0**
P0	2.000	1.000	1.000	4.000	0.800	1.000

Notice that it is a very small mistake; instead of 1.333, (s)he entered 1. So, the pairwise comparison matrix for Criterion 1 becomes:

Table 7. Example of Matrix A with an Incorrect Input

Criterion 1	**P1**	**P2**	**P3**	**P4**	**P5**	**P0**
P1	1.000	0.667	0.500	2.000	0.400	0.500
P2	1.500	1.000	0.750	3.000	0.600	0.750
P3	2.000	1.333	1.000	4.000	0.800	1.000
P4	0.500	0.333	0.250	1.000	0.200	0.250
P5	2.500	1.667	1.250	5.000	1.000	1.250
P0	2.000	1.000	1.000	4.000	0.800	1.000

Table 8. The Result of the Induced Matrix $\boldsymbol{C} = \boldsymbol{AA} - n\boldsymbol{A}$

Criterion 1	**P1**	**P2**	**P3**	**P4**	**P5**	**P0**
P1	0.001	-0.169	0.000	0.001	0.000	0.000
P2	0.000	-0.251	0.000	0.000	0.000	0.000
P3	-0.001	-0.332	0.000	-0.001	0.000	0.000
P4	0.000	-0.082	0.000	-0.001	0.000	0.000
P5	0.001	-0.419	0.000	0.001	0.000	0.000
P0	-0.500	1.333	-0.250	-1.000	-0.200	-0.250

One can easily verify that Step 3, i.e., $\boldsymbol{C} = \boldsymbol{AA} - 6\boldsymbol{A}$ will produce the following matrix, and following the procedure suggested by Ergu et.al. [30], the largest absolute value is 1.333, which belongs to the comparison between P0 and P2. It is easy to see that $a_{0,1} \times a_{1,2} = 2 \times 0.6667 = 1.3333 \neq a_{0,2} = 1$. Hence, the appraiser realizes the mistake made on the pairwise comparison between P0 and P2. This example should illustrate the benefit of the pairwise comparison approach together with a simple theorem adopted from Ergu et.al. [30]. Again, for practical application in a valuation company in Indonesia, we do not recommend going with an inconsistent matrix. We also believe that this approach is an extension of Direct Sales Comparison. Hence, there are typically only a handful of comparables (5 – 10 comparables), and the appraiser should try to find the source of the inconsistency and take corrective action. Furthermore, we also do not suggest to implementing the corrective action that is proposed by Ergu et.al. – it was a great academic proposal, but a bit complicated for practical application. Instead, we suggest finding the largest absolute value of matrix $\boldsymbol{C}$ as a starting point, and then use the transitivity property $a_{i,j} \times a_{j,k} = a_{i,k}\ \forall i, j, k$ to figure out the source of inconsistency. Of course, this example is an easy example to illustrate the process.

For the comparative analysis of the conventional vs. proposed method, we depict the advantages of the proposed hybrid AHP-QRVE method over the conventional approach. A comparative analysis is summarized based on the simulation of a appraiser's input error. Under the conventional Likert-scale QRVE, when an appraiser makes a subtle mistake - such as inputting an incorrect rating for Project 0 - the system processes the flawed data blindly without any quality control. This hidden error directly distorts the final output, causing the predicted market rent to drop erroneously to \$275. The conventional method provides no mechanism to alert the appraiser regarding this logical inconsistency. Otherwise, the proposed approach demonstrates an internal consistency-checking advantage over conventional QRVE through Theorem 2. As shown in Table 8, the exact location and magnitude of the appraiser's error (the value of 1.333) are immediately captured and flagged within the induced matrix $\boldsymbol{C}$. This programmatic alert prevents the valuation from proceeding with flawed data, guides the appraiser to perform necessary corrections using transitivity properties, and preserves the baseline estimate of $306.25. In this numerical simulation, the proposed method demonstrates an internal consistency-checking advantage over conventional QRVE by identifying an inconsistent pairwise judgment before the valuation process proceeds.

In practical applications, concerns may arise regarding the computational complexity of the proposed approach as the number of properties increases. Theoretically, evaluating n properties (subject plus comparables building) across m characteristics requires a total of pairwise comparisons due to the reciprocal nature of the matrix.

$$Number\ of\ Pairwise\ Comparison = m \times \frac{n(n-1)}{2} = m \times \frac{(c+1)c}{2} \tag{4}$$

For instance, an extreme scenario involving 19 comparable properties ($n = c + 1 = 20$) properties and $m = 5$ characteristics would mathematically demand 950 individual comparisons, which could induce decision fatigue for the appraiser. However, such a large dataset is rarely utilized in standard property appraisal practice, where a selection of 3 local comparables (the minimum number, yielding $n = 3 + 1 = 4$ total properties) is typically sufficient and compliant with industry standards [29] to focus on objects that are truly similar and comparable (homogeneous). And also, the assessment report for building valuation on that comparable number is considered valid and can be legally accounted for. A large amount of data does not guarantee accuracy if the characteristics are too broad. For a realistic valuation scenario utilizing 3 to 5 comparables, the total required comparisons drop dramatically to only 30 and 75, respectively, which can be easily completed within a few minutes. Furthermore, in actual valuation practice, this methodology is designed to be implemented via Computer-Aided Valuation (CAV) systems, such as automated spreadsheet templates or proprietary valuation software. Under this computerized setup, the appraiser is only required to input pairwise judgments in the lower-triangle cells. To streamline the evaluation process, the CAV system automatically populates the corresponding upper-triangle entries using the reciprocal property ($a_{ji} = 1 / a_{ij}$), and sets the diagonal elements to 1. Because the lower-triangle judgments are provided directly by the appraiser and may contain human errors or logical contradictions, the system subsequently deploys Theorem 2 ($C = AA - nA$) to verify matrix consistency and flag any inconsistent pairs for revision. Therefore, the integrated AHP-QRVE approach remains highly feasible, efficient, and practical for real-world valuation companies.

CONCLUSIONS

Theoretically, this study established a mathematical connection between conventional Likert-scale ratings and consistently reciprocal matrices, thereby extending the traditional QRVE framework with an internal consistency verification mechanism. By integrating the pairwise comparison method from AHP into the QRVE framework, this study bridges subjective appraiser scoring with rigorous verification that offers a direct solution to the inherent bias and logical inconsistencies found in ordinal scale evaluations. Rather than relying on unverified Likert inputs, the proposed approach enforces analytical rigor through the induced verification matrix $\boldsymbol{C} = \boldsymbol{AA} - \boldsymbol{nA}$, which the conventional QRVE method fails to capture. Numerical simulations demonstrate that this mechanism effectively detects and flags subtle human judgment errors that conventional QRVE methodologies fail to capture. Practically, the findings show that this methodology may assist property appraisal institutions in ensuring logical coherence in valuation reports.

Despite its advantages, this study is subject to certain limitations. First, the validation of the model relies primarily on controlled numerical simulations rather than an extensive large-scale empirical dataset. Second, as highlighted by the computational analysis, widespread adoption by practitioners still faces scalability challenges when the number of comparable objects increases significantly, which increases the required number of comparisons quadratically. To address these limitations, future research should focus on implementing the model for a wider empirical database across various property sectors to test its robustness against market anomalies. Additionally, the development of specialized computer-aided valuation (CAV) software beyond standard Microsoft Excel environments is crucial. Such software should automate the generation of upper-triangle matrices using transitivity properties, thereby reducing the appraiser's manual cognitive load and optimizing the transition toward consistency-based valuation in professional practice. Last, the proposed mechanism evaluates the internal consistency of pairwise judgments rather than their external accuracy. Therefore, a logically consistent matrix may still reflect inaccurate market assumptions or systematic appraiser bias.

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